RESEARCH OF DYNAMICS OF SPACE CABLE SYSTEMS STABILIZED BY ROTATION

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ABSTRACT

The analysis of opportunities of use of rotating space tethered systems (STS) is carried out and it is shown that their realization allows one to receive additional profits practically in all areas of possible application of STS. Peculiarities of dynamics of STS rotational motion are considered and basic problems of researches in their connection with problems of nonlinear dynamics are determined. A model of STS perturbed motion, suitable for researches of the dynamics by asimptotical methods of nonlinear mechanics is offered. The basic regularities of an evolution of STS rotational motion under influence of internal and external perturbations are considered: influence of essentially nonlinear internal oscillations on dynamics of the systems is determined; regularities of influence of the dissipative forces of aerodynamic resistance and the internal friction in a material of the tether are determined. Interconnection of orbital and relative motions is analysed and their control laws with the use of internal forces by periodic change of distance between bodies are offered.

1. OPPORTUNITIES OF APPLICATION OF ROTATING STS

The Canadian project BICEPS – BIstatic Canadian Experiment on Plasmas in Space [1] and its support by NASA [2] are represented as a significant step in development of a direction of use of rotating STS. An opportunity of rather simple changes of the tether length, the force of its tension and angular velocities of rotation of the system have made possible to offer the use rotating STS for space plasma research.

Rotation of STS in the project of an aerodynamic probe would make possible somewhat to dicrease velocity of motion of a probe in an atmosphere by means of inverse with respect to STS orbital rotation. Thus it is possible to provide that after immersing in a low atmosphere the tethered satellite making a rotary motion around the STS centre of mass will leave dense layers of an atmosphere and the rather long time will be in conditions favourable for cooling. These conditions can be essential for the equipment, for example carrying out supervision (a photographing) surfaces of the Earth. Rotating STS, passing in the rotation different layers of an atmosphere as though scans it. Such research of an atmosphere can be alternative to research by means of a radial STS of a kind of "beads" of sensors [3,4].

Use of STS rotating in an orbit significantly expands opportunity of their use for transport operations because significantly increases an accumulation by STS both kinetic energies and angular momentum.

The STS rotating around centre of mass can serve also as integrated sensor for research of influences of the Earth fields. When STS is accumulating on a half-revolution of the rotation around the centre of mass results of influence as change of the angular velocity of the rotation it allows one to receive an integrated, average estimation of a difference of influences on tethered bodies. So the project of the small STS consisting in rotation of small STS with two trial end bodies with different ballistic factors is interest for research of latitudinal changes of density of an atmosphere.

Small rotating STS can serve as the standard of length for calibration and measurements of the characteristics onboard and ground optical and radar-tracking systems.

The question of use the rotating electrodynamic STS is outside of attention of the researchers for today also. And though this question concerns more to physicists it seems that use of rotating STS in a magnetic field as Hertz's doublet will allow one to generate in a separate conductor an alternating current. The positive decision of this question would open a perspective opportunity for realization of the project of electrodynamic STS in vacuum i.e. without creation of a closed contour of a current in an ionosphere and appropriate equipment and on higher orbits.

The stabilization of motion of small STS by creating of its rather fast rotation around the centre of mass is simple and reliable and in many cases only possible passive way. Therefore rotating STS represent the special interest for the projects of creation of small cable systems which can find wide application for the experimental and scientific purposes and for creation of tethered microsatellite systems.

It is obviously that even brief consideration of opportunities of rotating STS shows that their use allows one to receive new effects practically in all areas of STS use.

2. RESEARCH PROBLEMS OF DYNAMICS OF ROTATING CABLE SYSTEMS

Dynamics of rotary motion of STS has a number of essential differences from dynamics of a radial STS the problems of which were widely investigated within the framework of preparation of the projects TSS-1 and TSS-2. Here a qualitatively different regime of motion – the regime of stabilization by rotation takes place. Basic questions of dynamics of a STS rotational motion are distinct also: how orientation of the plane of STS rotation will vary?; how its velocity of rotation will vary? Areas of resonance regimes of motion and their properties are distinct.

Low rigidity of bodies connection is one of the basic peculiarities of STS. By virtue of it and also by virtue of a unilateralness of action of cables the regimes of STS motion with large amplitude of oscillations on internal degrees of freedom are possible, the character of which is essentially nonlinear. And these oscillations despite of their dissipation by internal friction can constantly be excited in motion of STS with respect to, for example, thermal impact at an input-output of STS in a shadow of the Earth. Thus the research of dynamics of rotational motion is connected with the decision of the nonlinear mechanics problem of influence of essentially nonlinear oscillations of bodies on internal degrees of freedom on dynamics of systems in a central field of forces.

The analysis of works on this problem shows that to the present time the significant quantity of works on dynamics of systems of connected bodies with oscillatory parts exists. However the majority of researches is carried out in the assumption of a smallness of amplitudes and quasistaticness of system oscillations caused by final rigidity of connections. These assumptions correspond either linear oscillations of a system on internal degrees of freedom or in general exclude from consideration influence of own elastic oscillations of the system. For problems in such statement to the present time the well developed techniques of researches exist. The works on dynamics of systems at oscillations of bodies on internal degrees of freedom with large amplitude and at a getting off bodies from connection to the present time are practically away. Developed techniques of dynamics research of such systems are away also.

The large extent of STS causes essential increase of forces and moments influencing to its motion. A correctness of a consideration of STS motion about an orbit of the centre of mass within the framework of limited statement of a problem should each time prove to be true. At the same time, the research problem of long-duration STS rotation in an orbit requires the careful analysis of influence both external environment and properties of connection as long influence even of small forces can result in significant deviations of motion from programm. Thus research of dynamics of STS rotational motion is connected with the solution of the problem of evolution of motion of extended systems in earth orbits.

Because this problem had not direct practical importance earlier, many its aspects have remained not investigated. For example, the analysis of interrelation of orbital and relative motions in a Newtonian field of forces is not completed. There are not enough works on research of possibilities of control of motion of connected bodies systems in a Newtonian field of forces by way of redistribution with the help of internal forces of an angular momentum between orbital and relative motion.

Named problems of nonlinear dynamics of space systems include:

- the problem of redistribution of energy in resonance regimes;

- the problem of synchronization and stochastization of motions.

The analysis of a state of named problems for today shows that their research first of all requires the construction of a qualitative picture of nonlinear dynamics, i.e. definition of the basic regularities and possible effects of nonlinear interactions. With this purpose use of model problems is expedient, i.e. use of such simplified models of dynamics in which essential interactions of researched process are kept only.

3. MODEL OF STS PERTURBED MOTION

We shall consider motion of system of two dot masses connected by an elastic weightless flexible tether in a central Newtonian field of forces. The equations of motion of considered system are

$$\begin{aligned} m_1\ddot{\vec{R}}_1 &= -\frac{\mu m_1 \vec{R}_1}{{R_1}^3} + T_1\vec{e_r} + \vec{F}_1, \\ m_2\ddot{\vec{R}}_2 &= -\frac{\mu m_2 \vec{R}_2}{{R_2}^3} - T_1\vec{e_r} + \vec{F}_2, \end{aligned} \quad (1)$$

where m_i are the masses of material points, $\vec{R}_i$ are their radiuses-vectors from the Newtonian attracting centre, $T_1\vec{e_r}$ is the force, acting along the connection line (the elastic force of the tether), $\vec{e_r}$ is a unit vector directed along the connection line, $\vec{F}_i$ is the total vector of other forces, acting on i-th body ($i = 1, 2$), μ is the gravitational constant.

From (1) we obtain the equation of relative motion and the equation of centre of mass motion of the system

$$\ddot{\vec{r}} = \ddot{\vec{R}}_2 - \ddot{\vec{R}}_1 = -T\vec{e_r} + \vec{F}, \quad \ddot{\vec{R}} = -\frac{\mu\vec{R}}{R^3} + \vec{F}^*, \quad (2)$$

where $\vec{R} = \dfrac{\vec{R}_1 m_1 + \vec{R}_2 m_2}{M}$ is the radius-vector of the center of mass of the system from the attractive center , $M = m_1 + m_2$,

$$T = T_1\frac{M}{m_1 m_2}, \quad \vec{F} = \frac{\vec{F}_2}{m_2} - \frac{\vec{F}_1}{m_2} + \vec{F}_{gr},$$

$$\vec{F}^* = (\vec{F}_1 + \vec{F}_2)/M + \vec{F}^*_{gr},$$

$$\vec{F}_{gr} = \mu(\vec{R}_1/R_1^3 - \vec{R}_2/R_2^3),$$

$$\vec{F}^*_{gr} = \mu\vec{R}/R^3 - \frac{1}{M}\sum_{i=1}^{2}\mu m_i \vec{R}_i/R_i^3.$$

It is supposed that the ratio of the system length $r = |\vec{r}|$ to distance from the centre of mass up to the attracting centre R is a small value $r/R \ll \varepsilon_1$, and also a smallness of the disturbing accelerations $\vec{F}_i$ in the sense that kinetic energies of motion of the system around the centre of mass and motion of its centre of mass essentially surpass work of the appropriate disturbing forces on a considered interval of time.

We shall introduse the right coordinate systems. $C\xi\eta\zeta$ is an absolute coordinate system with the origin in the attracting centre C. $CXYZ$ is a "perigee" coordinate system connected with an instant orbit of the centre of mass motion. The axis CX is directed from C to the pericenter of an orbit, the axis CZ is derected on the vector of an angular momentum of the centre of mass motion (binormal). $Oxyz$ is a moving coordinate system with the origin in the centre of mass of the system. The axis Oz is directed on the vector of an angular momentum of relative motion, axis Ox is derected on $\vec{r}$. Mutual orientation $C\xi\eta\zeta$ and $CXYZ$, $CXYZ$ and $Oxyz$ is defined by Eulerian angles Ω, i, ω_π and ψ, θ, ϕ accordingly.

On the basis of the theorem of change of an angular momentum and using known relations between derivatives of the angles and components of angular velocity we obtain [5]

$$\dot{\psi} = \frac{rF_3 \sin\varphi}{L\sin\theta} - \omega_\psi, \quad \dot{\theta} = \frac{rF_3\cos\varphi}{L} + \omega_\theta,$$
$$\dot{L} = rF_\varphi, \quad \dot{\varphi} = \frac{L}{r^2} - \dot{\gamma}\cos\theta + \omega_\varphi, \tag{3}$$
$$\omega_\psi = \cot\theta\left(\frac{di}{dt}\sin\gamma - \dot{\Omega}\sin i\cos\gamma\right) - \dot{\Omega}\cos i -$$
$$-\dot{\omega}_\pi, \quad \omega_\theta = \dot{\Omega}\sin i\sin\gamma + \frac{di}{dt}\cos\gamma,$$
$$\omega_\varphi = \dot{\Omega}\left(\sin\theta\sin i\cos\gamma - \cos\theta\cos i\right) -$$
$$-\frac{di}{dt}\sin\theta\sin\gamma.$$

where $L = \left(\vec{r}\times\dot{\vec{r}}, \vec{e}_3\right)$ is the magnitude of a specific angular momentum of the system relative motion, $\vec{e}_3$ is a unit vector of the axis Oz, F_2, F_3 are the projections $\vec{F}$ on the axes Oy, Oz accordingly, $\gamma = \omega_\pi + \psi$.

The equation of change r we obtain by way of projecting the first of the equations (2) on the axis Ox

$$\ddot{r} - \frac{L^2}{r^3} = -T + F_1, \tag{4}$$

where F_1 is the projection $\vec{F}$ on the axis Ox.

The equations (3), (4) and the equations of perturbed Kepplerian motion

$$\frac{di}{dt} = \frac{R}{p}\cos u\widetilde{F}_{03}, \ \dot{\Omega} = \frac{R}{p}\frac{\sin u}{\sin i}\widetilde{F}_{03}, \ \dot{p} = 2R\widetilde{F}_{02}$$

$$\dot{e} = \widetilde{F}_{01}\sin\nu + \left[\cos\nu + (e+\cos\nu)\frac{R}{p}\right]\widetilde{F}_{02}, \tag{5}$$
$$\dot{\omega}_\pi = -\frac{\widetilde{F}_{01}\cos\nu}{e} + \widetilde{F}_{02}\left(1 + \frac{R}{p}\right)\frac{\sin\nu}{e} -$$
$$-\frac{R}{p}\sin u\cot i\widetilde{F}_{03}, \quad \dot{u} = \frac{\sqrt{\mu p}}{R^2} - \frac{R}{p}\sin u\cot i\widetilde{F}_{03}.$$

where $\nu = u - \omega_\pi$, $R = p/\left(1 + e\cos\nu\right)$, $\widetilde{F}_{0i} = F^*_{oi}\sqrt{p/\mu}(i = 1,2,3)$, F^*_{oi} are the projection F^* on the axes CX, CY, CZ accordingly, e is the eccentricity, p is the focal parameter, ν is the true anomaly make complete system of the equations of motion.

The basic peculiarity of these equations is the presence of the nonlinear oscillatory link (4), describing longitudinal oscillations of the system on an internal degree of freedom (the oscillations of distance between bodies). The technique of research of such systems is given in [6]. Idea of a deriving of the equations of perturbed motion of the oscillatory link is following. We shall assume that the elastic force of the tether is those that in unperturbed motion that is at $F_1 = 0$ and $L = const$, r periodically changes between extreme meanings r_1 and r_2, $r_1 < r_2$. Then in the unperturbed motion r can be presented in form

$$r = a - b\Phi\left(w(t)\right), \tag{6}$$
$$a = \frac{r_1 + r_2}{2}, \quad b = \frac{r_2 - r_1}{2}.$$

If to accept that $\dot{w} = k = const$ then the form of the oscillations will be described by the function $\Phi(\cdot)$. If to fix the form of the oscillations in the unperturbed motion, i.e. assuming that $\Phi(\cdot)$ is an invariable function and considering b and w as new variable, we obtain the equations of the perturbed motion of the longitudinal oscillations

$$\dot{b} = \left\{-kbF_1\frac{d\Phi}{dw} - \dot{L}\left[L\left(\frac{1}{(a+b)^2} - \frac{1}{r^2}\right) + \right.\right.$$
$$\left.\left. + \frac{\partial a}{\partial L}\frac{\partial V(a+b)}{\partial a}\right]\right\}\left[\frac{\partial V(a+b)}{\partial a}\left(1 + \frac{\partial a}{\partial b}\right)\right]^{-1}, \tag{7}$$
$$\dot{w} = k - \left(\dot{b}\Phi(w) - \dot{b}\frac{\partial a}{\partial b} - \frac{\partial a}{\partial L}\dot{L}\right) / \left(b\frac{d\Phi}{dw}\right),$$

where

$$V(r) = \int T dr + 0.5L^2/r^2,$$

a, b and L are connected by a rilation $V(a+b) = V(a-b)$.

The equations system (3), (5), (7) is suitable for its research by asimptotical methods of mechanics. In a general case of rather fast rotation of the system about the centre of mass the system equations has two fast variables φ and w and two small parameters $r/R = \varepsilon_1$ and $(\dot{u}/\dot{\varphi})^2 = \varepsilon_2$. We note that for a regime of motion with large amplitude of the longitudinal oscillations when b/a is not small the dependence $\dot{\varphi}$ from w complicates direct application of the averaging method. In this case expediently to use angular value $\varphi_1, \dot{\varphi}_1 = L/r^2$ as a new independent variable. The scheme of a deriving of the equations of the perturbed motion in this case does not change[6].

4. BASIC RESULTS OF RESEARCHES OF NONRESONANCE REGIMES OF MOTION

The research of system dynamics was carried out by the averaging method. The equations of the first approximation on small parameters ε_1 and ε_2 are constructed and on their basis the following conclusions are made.

4.1 Influence of longitudinal oscillations

The longitudinal oscillations do not qualitatively change the nature of the evolution of the parameters of system motion and the evolution of its motion in the first approximation coincides with the evolution of motion of a dumb-bell with the certain length of a bar. In a case of essentially nonlinear longitudinal oscillations and in particular in a regime of motion with a getting off from the connection the length of this bar is equal $\left({r_*}^4/{r^*}^2\right)^{1/2}$

$$
\begin{aligned}
{r_*}^4 &= \frac{1}{2\pi}\int_0^{2\pi} \frac{dw_1}{\left(a_1 - b_1\Phi_1(w_1)\right)^4}, \\
{r^*}^2 &= \frac{1}{2\pi}\int_0^{2\pi} \frac{dw_1}{\left(a_1 - b_1\Phi_1(w_1)\right)^2},
\end{aligned}
\qquad (8)
$$

where a_1, b_1, Φ_1, w_1 are defined similarly a, b, Φ, w in (6) [6]:

$$\frac{1}{r} = a_1 - b_1\Phi_1(w_1), \; \frac{dw_1}{d\varphi_1} = const.$$

In case of oscillations with small amplitude $b/a \ll 1$ the evolution of the system rotational motion coincides with evolution of motion of a dumb-bell with the length of a bar r_0 where r_0 is defined by equality of elastic and centrifugal forces.

4.2 Effect of aerodynamic forces

The aerodynamic influences on the system are described by forces acted on each body of the system

$$\vec{F}_{ai} = -\dot{\vec{R}}_i|\dot{\vec{R}}_i|k_i m_i, \qquad (9)$$

where $k_i = \rho_i c_{xi} S_i/2m_i$, ρ_i is the density of an atmosphere, S_i is the area of midship, c_{xi} is the aerodynamic factor of resistance $(i = 1, 2)$. Then the influence on relative motion of the system with accuracy to the members about $k_i|\dot{\vec{r}}|$ looks like

$$\vec{F}_a = (k_1 - k_2)|\dot{\vec{R}}|\dot{\vec{R}} - I\left[|\dot{\vec{R}}|\dot{\vec{r}} + (\dot{\vec{R}}, \dot{\vec{r}})\dot{\vec{R}}/|\dot{\vec{R}}|\right],$$

$$I = (k_2 m_1 + k_1 m_2)/M \qquad (10)$$

The second member of the formula (10) gives the dissipative component of aerodynamic accelerations.

Only dissipative component of aerodynamic forces influences on the evolution of the system motion [7]. Their conservative component results only to small almost periodic oscillations of the system.

At influence only of aerodynamic forces the angular momentum of the system relative motion aspires to be situated in the orbit plane (the plane of the system rotation aspires to lie perpendicularly the plane of the orbit). This effect takes place for any orbit of the centre of mass. And in each moment of time the system aspires to rotate perpendicularly to the vector of velocity of the centre of mass that corresponds to tendency to a state giving a minimum velocity of a energy dissipation in the relative motion.

This tendency of the system to avoid friction results in that the angular momentum of the relative motion of the system, making precession about the normal to the orbit plane under influence of the Newtonian field of forces aspires to lie in the orbit plane under influence of resistance of environment. For elliptic orbits the system aspires to a unique state corresponding to integrated minimum of the energy dissipation (more often this state corresponds to rotation of the system perpendicularly to the tangent to the orbit in its perigee). Thus under action of aerodynamic forces the system aspires to the state of the least aerodynamic dissipation the relative motion energy .

4.3 Influence of energy dissipation in material of the tether

In the first approximation on small parameters an internal dissipation of energy results only in monotonous damping of the own elastic oscillations and does not change qualitatively the motion evolution of the system. Therefore the influence of the energy dissipation on the evolution of motion (the stage of slow evolution) for the majority of real space systems is neglectly small. Nevertheless, the question of evolution of extended visco-elastic systems in the Newtonian field of forces is interest for celestial mechanics and for definition of general regularities of motion and is a constant subject of researches and discussions (see for example [10]). The used the elementary visco-elastic system of two material points has allowed us to carry out the deep analysis of the relative motion and to consider general regularities of the translational-rotary motion.

The motion of the system of two material points connected by a weightless visco-elastic tether in the Newtonian field of forces was considered. The elastic properties of tether are described by the Hook's law and dissipative properties described by introduction of equivalent "viscous friction" :

$$\ddot{\vec{r}} = -(c_m(r-d) + \varsigma\dot{r})\frac{\vec{r}}{r} + \vec{F}_{gr},$$

$$c_m = c\left(\frac{1}{m_1} + \frac{1}{m_2}\right),$$

where c is the coefficient of rigidity of the tether, d is its nominal length, ς is the factor describing viscous friction in the tether. Research of the influence of the energy dissipation in a material of the tether is carried out with accuracy to the second order of the smallness inclusive.

In a case when $(r/R)^2$ neglectly small ($\varepsilon_1^2 \ll \varepsilon_2^2$) the trajectory of the system centre of mass can be considered as an unperturbed Kepplerian orbit, i.e. the problem can be considered in limited statement. In this case under influence of the energy dissipation in the tether the system in each moment of time aspires to the location giving the minimum of the dissipation of energy of the relative motion that corresponds to aspiration of the system to rotate perpendicularly $\vec{R}$. For any orbit of the centre of mass the angular momentum of the relative motion aspires to lie in the orbit plane ($\theta \to \pi/2$). And for θ close to $\pi/2$ the action of the dissipative forces traces an elliptic shape of the orbit and aspires to arrange the plane of the system rotation perpendicularly to the radius-vector of the orbit pericenter.

Comparison of received results with conclusions about influence dissipative component of aerodynamic forces allows us to assume that influence of dissipative forces of a various physical nature on the relative motion of the system in the Newtonian field of forces results in aspiration of the system to a location appropriate to the least loss of energy of the relative motion. This assumption corresponds to known hypothesis [9] about aspiration of material systems to avoid friction. However in [9] this hypothesis is considered as the resulting tendency of systems motion : in result of influence of dissipative forces the motion velocities causing of energy dissipation become equal zero. In considered cases the system in each moment of time aspires to the location appropriate to the minimum of energy dissipation and action of dissipative forces is directed on change of all parameters of system motion according to this principle. Thus it is possible to put forward the assumption that aspiration of systems to avoid friction is tendency acting in each moment of time, and also that the influence of dissipative forces is directed on change of motion parameters according to this tendency.

In the translational-rotary motion of the system the action of dissipative forces is directed on the following changes in the system motion: the absolute value of the angular momentum of the system relative motion decreases, being redistributed in the angular momentum of the orbital motion, the eccentricity of the orbit ($e_0 \neq 0$) grows, the angle of a nutation θ aspires to some value $\pi/2-\alpha_c$, $0 < \alpha_c < \pi/2$ and only in the end aspires to $\pi/2$, i.e. the influence of dissipative forces aspires to transfer the backward rotation $\theta > \pi/2$ systems in direct $\theta < \pi/2$. The interpretation of obtained results from the point of view of the formulated above assumption has brought interesting results. The fact is that increase of the eccentricity of the orbit and the system aspiration to direct rotation, generally speaking, are directed on increase of capacity of dissipative forces, i.e. the change of these parameters cannot be explained within the framework of the introduced assumption. The analysis of change of the system evolution in comparison with the motion of the system about of a unperturbed orbit has allowed us to make a conclusion that the character of these changes corresponds to aspiration of the orbital motion to reduce a loss of energy (to increase its reception for) its motion.

The general picture of the evolution of the system motion under action of the dissipative forces combines of aspiration of each of motions – the orbital and the relative – to reduce a loss of energy (to increase its reception for) its motion.

Certainly, it is only a special case. But the contradiction between the different forms of motions in their aspiration to keep or to get energy is represented rather interesting, as such contradictions leave a chance for development of motion, and not just for its trivial forms.

4.4 Interrelation of translational and rotary motions

We consider motion of the system in the Newtonian field of forces at absence of other external forces $\vec{F_i} \equiv 0$. The constant angular momentum of the system $\vec{C}$ is equal to the sum of angular momentums of orbital and relative motions

$$M\sqrt{p\mu}\vec{e_3^*} + \frac{m_1 m_2}{M} L\vec{e}_3 = \vec{C}, \qquad (11)$$

where $\vec{e_3^*}$ is a unit vector of the axis CZ.

Projections of the equation (11) on the axes of the coordinate system $C\xi\eta\zeta$, where the axis $C\zeta$ is directed on the vector $\vec{C}$ give three integrals of the areas:

$$\omega_\pi = \pi - \psi,$$

$$\sqrt{p\mu}\cos i + \frac{m_1 m_2}{M^2} L\cos(\theta - i) = C', \qquad (12)$$

$$\frac{m_1 m_2}{M^2}\frac{L}{\sin i} = \frac{\sqrt{p\mu}}{\sin(\theta - i)} = \frac{C'}{\sin\theta)}, \; C' = \frac{|\vec{C}|}{M}.$$

The second and third equality (12) are the relations of a triangle, formed by the vectors of angular momentums of motions. The first equality (12) follows from the costruction of the coordinate systems and means that the axis of the system rotation about the centre of mass lays in a plane normal to a line of nodes of the orbit that coincides with the third generalized Cassini law [10].

From (12) it is follows that correctness of limited statement of the problem , i.e. the assumption of independence of the orbital motion from relative one requires not only fulfilment of the condition $r/R \ll 1$ but also smallness of the ratio of the module of the angular momentum of the relative motion to the module of the angular momentum of the orbital motion: this ratio should be value of the higher order of a smallness than taken into account perturbing influences. In the considered case of gravitational influences a condition

$$\frac{m_1 m_2}{M^2}\frac{L}{\sqrt{p\mu}} \,/\, \frac{r}{R} \sim \frac{m_1 m_2}{M^2}\frac{r}{R}\frac{\omega}{\omega_0} \ll 1$$

must be carried out where ω_0 and ω are the magnitudes of the angular velocities of orbital and relative motions.

The basic evolutionary effect of the motion of the system quickly rotating around the centre of mass consists in rotation of the plane formed by the vectors of angular momentums, around of the total angular momentum [11]. In this case one of component angular velocity of this rotation does not depend on both masses of bodies and linear sizes of the system, and it is proportional to the relation of angular velocities of orbital and relative motions.

5. USE OF RESONANCE REGIMES FOR CONTROL OF MOTION

We consider motion of the system in the Newtonian field of forces at absence of other external forces $\vec{F_i} \equiv 0$. Distance between bodies we shall consider as some known function from angles of the system orientation, and r periodically changes near some value a with amplitude b and can be submitted as $r = a - b\cos w$ where a phase of oscillations w is a function from angles of the system orientation. The equation (4) allows one to determine value of necessary control along a line of the tether for fulfilment of this functional dependence.

5.1 Control of the relative motion

Possibilities of control for the relative motion we consider in limited statement of the problem. For an estimation of change of angular velocity of the relative rotation we shall consider a motion in the plane of a circular orbit. The equations (3) will accept a form

$$\dot{L} = -\frac{3}{2}\frac{\mu}{R^3}\sin 2\alpha,\ \dot{\alpha} = \frac{L}{r^2} - \dot{\nu},\ \dot{\nu} = \sqrt{\frac{\mu}{R^3}},\quad (13)$$

where $\alpha = \varphi + \psi - \nu$ is the angle between $\vec{R}$ and $\vec{r}$. The largest increase L at a rotational motion will give the law of regulation of the tether length in a form $r = a - b sign(\sin 2\alpha)$. The transition from an initial librational motion to rotary is realized by a swinging of the system (the swings), for example by the law $r = a - b sign(\alpha' \sin 2\alpha)$ where the stroke designates derivative on ν. Below for a circular orbit of radius $R = 6671 km$ and for the initial conditions $\alpha'_0 = 10^{-5}, \alpha_0 = \pi/2$ for the law of control $r = a\,(1 - 0.1 sign(\sin 2\alpha))$ estimations of achievement by one of bodies of system ($m_1 = m_2$) of velocity sufficient for transition to a hyperbolic trajectory at break of the tether are resulted [11].

$a,\ km$	α'	Time, day	$T, m/s^2$
1	5526	173,3	41015
50	110	3,4	827,1
200	27	0,8	210,5
500	10,1	0,3	82,7

In the last column values of centrifugal acceleration appropriate to angular velocity are brought. The parameters of transition can be improved for system of bodies of different mass and at motion on an elliptic orbit [12]. The motion of the second body after break of the tether will depend on distance up to the centre of mass. In particular, it is possible to select such parameters of the system that the second body remained practically on the same orbit but rotated in the opposite direction.

For control of value of an angle of a nutation θ two simple laws of control of the tether length based on the tuning of the system oscillations in an internal resonance $r = a - b/sin 2\varphi$ and external resonance $r = a - b\sin 2(\nu - \psi)$ of system were considered. It was shown that both laws allow one to operate value of the angle θ. The change of the angle of a precession does not require a control.

5.2 Control of the orbital motion

The possibility of control of parameters of the orbital motion of the system follows from fact that at orientation of the system different from equilibrum the attraction force have transversal and normal components. On the basis of this fact the various schemes of control for elements of an orbit with the use of internal forces can be constructed.

The possibility of change of orientation of the orbit plane follows from an possibility of change of value and orientation of the angular momentum of yhe relative motion. As the ratio of velocity of change L to angular orbital velocity proportionally $\sqrt{\mu/p^3}r^2/L$, by virtue of (12) the essential change of the inclination of the orbit i is possible during $(p/r)^2/2\pi$ of revolutions on the orbit. Velocities of control for the value of the angle Ω are similar as the angular momentum of the centre of mass motion moves on a surface of a cone with an angle of a half-solution equal i:

$$\sin i = \frac{m_1 m_2}{M^2}\frac{L\sin\theta}{C'}.$$

The change of the argument of the pericenter and the eccentricity of the orbit is possible without change of the module and orientation of the angular momentum of the orbital motion [12,13].

We shall consider possibilities of change of the focal parameter at a motion of the system in the plane of the orbit. The equation (3), (5) in this case can be written as

$$\dot L = -\frac{3}{2}\frac{\mu}{R^3}r^2\sin 2\alpha, \quad \dot\alpha = \frac{L}{r^2} - \dot u,$$

$$\dot p = 3\frac{m_1 m_2}{M^2}\sqrt{\frac{p}{\mu}}\frac{\mu}{R^3}\sin 2\alpha, \quad \dot u = \frac{\sqrt{\mu p}}{R^2},$$

$$\dot e = \frac{3}{2}\frac{m_1 m_2}{M^2}\frac{\sqrt{p\mu}}{R^2}\left(\frac{r}{R}\right)^2\Big[\sin\nu(1-3\cos^2\alpha)+ \left(\cos\nu + \frac{e+\cos\nu}{1+\cos\nu}\right)\sin 2\alpha\Big], \tag{14}$$

$$\dot\omega_\pi = \frac{3}{2}\frac{1}{e}\frac{m_1 m_2}{M^2}\frac{\sqrt{p\mu}}{R^2}\left(\frac{r}{R}\right)^2\Big[\cos\nu(3\cos^2\alpha - 1) + \left(1+\frac{1}{1+\cos\nu}\right)\sin\nu\sin 2\alpha\Big].$$

From the equations (14) it is obviously that the greatest velocity of increase of parameter p is reached at $m_1 = m_2$ and $\alpha = 45^\circ$. Hence the dumb-bell with equal masses supported in a state $\alpha = 45^\circ$ at the expense of control moment created by internal forces (for example, fly-wheel placed in centre of masses) is the most effective model for increase of focal parameter. The solution of the equations (14) averaged on u looks like

$$p = \left[p_0^2 + \frac{3}{2}r^2(u-u_0)\right]^{1/2},$$

$$\omega_\pi = \omega_{\pi 0} + \frac{1}{4}\ln\left(\frac{p}{p_0}\right), \quad e = e_0\left(\frac{p}{p_0}\right)^{5/4}, \tag{15}$$

$$L = L_0 - 4(\sqrt{\mu p} - \sqrt{\mu p_0}),$$

where the value L characterizes change of an angular momentum of a fly-wheel. The estimations of an possibility of the given model of change of the orbit parameters for 100 revolutions around of the attracting centre depending on the length of a bar for $p_0 = 6671 km$ are given below.

r, km	$p - p_0$, km	e/e_0
0,1	0,00	1,000
1	0,07	1,001
10	7,06	1,011
100	672,5	1,101
200	2395,5	1,359

It is obviously from the table that the essential change of parameters of an orbit can be reached at lengths of the bar beginning with tens kilometers that essentially limits possibilities of practical realization of the given model of control. Uses of cables allows one considerably increase extent of the systems but complicates transfer of control moment. The idea of control of the orbit parameters of STS consists in fact that at the expense of internal forces distance between bodies changes and by that way angular velocity of rotation of system is adjusted so that in the necessary orientation the system was longer, than in a state giving an opposite effect of control . So for increase p the change of length of the rotating system can be realised under the law $r = a + b\sin 2\alpha$.

The solution of the equations (14) averaged on α and u differs from the solutions (15) in this case by that in the formulas for L, p, e r_p^2 will be instead of r^2 , and r_ω^2 – for ω_π, where

$$r_p^2 = \frac{a^3 b + 3/4ab^3}{a^2 + b^2/2}, \qquad r_\omega^2 = \frac{a^4 + 3a^2b^2 + 3/8b^4}{a^2+b^2/2}. \tag{16}$$

6. CONCLUSION

The considered opportunities of use of Space cable systems stabilized by rotation show perspectiveness of this direction. The research of dynamics of such systems is a complex problem connected with the solution a number of fundamental problems of nonlinear mechanics and in particular with the problem of influence of oscillations of bodies on internal degrees of freedom on dynamics of system.

Use of model problems for research of complex dynamic systems when in mathematical model essential elements to the researched phenomenon are allocated and kept only is convenient and in many cases only possible way of development of representations, definition and analysis of laws in their causal-consecuensal interrelation. In the considered model problem such important elements of dynamics of the real STS as an possibility of essentially nonlinear oscillations on internal degrees of freedom and large extent of system are kept. The considered model of rotational motion of the STS given by the equations (3) - (5) is close to a problem of three bodies in celestial mechanics. The distinction is that the elastic force of the tether is distinct from Newtonian force of gravitation. This difference has required development of new methods of researches.

Spent researches have allowed us to define the basic regularities of evolution of parameters under influence of external and internal perturbations, interrelation of orbital and relative motions and to consider basic possibilities of control for motion by way of resonance change of the tether length. The picture of possible regimes of motion of the considered model problem should be complemented by regimes of chaotic and synchronous motions the research of which will be carried out at the moment. Developed techniques and received results (some of which are unexpected), we hope, represent general interest for nonlinear dynamics problems.

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