**Pitch and Roll Attitude Maneuver of Twin-Satellite Systems Through Short Tethers**

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**Nomenclature**

- $a_i$ = plane of tether offsets for points $A_i$, $B_i$, $C_i$, and $D_i$, satellite $i$ from mass-center $S_i$; $i = 1, 2$
- $b_i$ = common vertical offsets (y, z coordinates) of satellite points $A_i$, $B_i$, $C_i$, and $D_i$; $i = 1, 2$
- $C = E A (m_1/L_{ref}, L^2)$, TSS rigidity parameter, where $E = $ Earth’s spin rate
- $f$ = tether modulus of rigidity, N
- $g$ = ratios of satellite masses ($m_1/m_2$), moments of inertia ($I_{11}/I_{22}$)
- $h_i(x, y, ..., y) = $ functions of $x, y, ..., y$; $k = 1, 2, ..., 9$
- $K_p = $ inertia parameters, $p = 1, 2, 3$
- $L_{1j}, L_{2j} = $ stretched and controlled unstretched length of $j$th tether, $L_j = L_{j0}(1 + \epsilon_j)$, m
- $L_{10}, L_{20} = $ nominal lengths of four tethers, m and reference length given by ($I_{11}/m_2$)$^{1/2}$, m
- $l, l_{1j}, l_{2j}, l_{10} = $ $l/L_{ref}, L/L_{ref}, L_j/L_{ref}, L_{10}/L_{ref}$, respectively
- $m_i, I_{i1} = $ mass, kg, and principal moment of inertia for satellite $i$ about $u_i$ axis; $u = x, y, z$, kg m$^2$
- $q_{1j}, q_{2j}, q_3 = $ pitch($\theta_j$), yaw($\psi_j$), and roll($\phi_j$), respectively, for satellite platform 1, deg
- $q_{4j}, q_{5j}, q_6 = $ pitch($\theta_j$), yaw($\psi_j$), and roll($\phi_j$), respectively, for satellite platform 2, deg
- $U(\epsilon_j) = $ 1 for $\epsilon_j \geq 0$ and 0 for $\epsilon_j < 0$
- $\beta, \eta = $ relative in-plane (pitch) and out-of-plane (roll) swing angles of $L$, deg
- $\delta_i = $ tether strains in the $i$th tether
- $\phi = $ true anomaly as measured from the reference line
- $\phi(\delta) = (\delta)/L_{ref}$
- $\phi(\delta) = (\delta)$ for jth tether; $j = 1, 2, ..., 4$ for $A_1 = A_2, A_3 = B_2, A_4 = D_1$, respectively
- $\phi(\delta)_{max} = (\delta)$ at $0$ true anomaly = 0 and maximum amplitude of $\phi(\delta)$
- $\phi(\delta), \phi(\phi) = (\delta)/d\delta$ and $d^2/\phi(\delta)$, respectively

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**Introduction**

SEVERAL investigators have proposed the use of tether(s) through active feedback control of their tension or offsets, alone or in combination. More recently, the authors have established the feasibility of using a multitethered auxiliary mass to augment the stabilizing effect of gravity gradient moments and hence ensure the desired fixed satellite orientation. Later, simple open-loop tether length control laws were developed to utilize the tether tensions for satellite attitude maneuver. In a related subsequent development the authors proposed splitting of the spacecraft into two halves and suitably connecting them through tethers so that each of the two halves plays the role of an auxiliary mass stabilizing the other, and as a consequence a high degree of pointing stability is ensured for the whole system.

Another related investigation demonstrated how the tether length control in dual tethered satellite systems based on judiciously developed control laws could be made use of for a very specific satellite pitch maneuver. Here, it is proposed to generalize this investigation by exploring the feasibility for achieving fixed or continual change slew maneuver of lower satellite or platform in pitch and/or roll modes while the other one is held along the nominal local vertical. For this, the model proposed (Fig. 1) comprises two satellites connected through four identical tethers in an inverted parachute-like configuration.

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**Fig. 1** Geometry of dual-tethered satellite system.
Equations of Motion

The Lagrangian formulation approach with Lagrange multipliers has been adopted to obtain the governing equations of motion for the constrained system. For brevity these equations taken from Ref. 9 are presented in concise symbolic form as follows:

\[ h_7(\beta', \beta, \beta, \eta, l, l') = \left( 1 + \frac{1}{f^2} \right) \frac{1}{1 - \cos \eta} \sum_{j=1}^{4} [\epsilon_j U(\epsilon_j)] \frac{\partial f_j}{\partial \beta} = 0 \]

\[ h_8(\eta', \eta, \beta', l, l') = \left( 1 + \frac{1}{f} \right) \frac{1}{(1/f)^2} \sum_{j=1}^{4} [\epsilon_j U(\epsilon_j)] \frac{\partial f_j}{\partial \eta} = 0 \]

\[ h_9(\ell', l, \beta, \beta, \eta, \eta) = \left( 1 + \frac{1}{f} \right) \frac{1}{1 - \cos \eta} \sum_{j=1}^{4} [\epsilon_j U(\epsilon_j)] \frac{\partial f_j}{\partial \ell} = 0 \]

subject to the constraint relations

\[ f_j = f_j(\ell, \theta, \phi, \eta, \beta, \eta, l, l) = 0 \]

\[ (j = 1, 2, 3, 4; i = 1, 2; k = 1, 2, \ldots, 6) \]

Synthesis of Open-Loop Control Laws

The governing complex nonlinear set of equations of motion is first examined for its feasible equilibrium configurations and their likely dependence on system parameters. For the twin-satellite system positioned in a fixed arbitrary gravity gradient configuration given by, e.g., \( \alpha, \beta, \eta, l \), a substitution of their equilibrium values and zero for their first and second derivatives into Eqs. (1) and (2) and carrying out a series of steps as outlined in Ref. 9, we can show that for achieving the desired orientation the tether lengths must be regulated according to the relations

\[ l_{\alpha i} = l_0 - (1/j) \frac{\sin \alpha_i - \alpha_i}{l_0}, \quad j = 1, 2 \]

\[ l_{\beta i} = l_0 - (1/2) \frac{\sin \beta_i - \beta_i}{l_0}, \quad j = 3, 4 \]

where

\[ \beta = \frac{1}{f} \sin^{-1} \left\{ \left( 1 + \frac{1}{f} \right) \frac{1}{(1/f)^2} K_1 \sin 2\phi_1 \right\} \]

\[ \eta = \frac{1}{f} \sin^{-1} \left\{ \left( 1 + \frac{1}{f} \right) \frac{1}{(1/f)^2} K_1 \right\} \]

For preliminary design estimates of tether lengths needed, the linearized perturbation pitching stability analysis was undertaken. The study was facilitated through sacrifice of rigor by way of several simplifying assumptions, e.g., \( g = 1 \), and \( K_2 = 1 \) corresponding to the unstable gravity gradient platform configurations. The stability criterion thus obtained can be written as

\[ L_{\alpha i} > (1 + 1/f) L_{rel}^2 \]

Even during stable attitude maneuvers, the transient disturbances are inherently present. It is easy to see that these can be minimized if the law regulating the equilibrium pitch or roll orientations is chosen as follows:

\[ (\alpha_1) = (\alpha_1) + [(\alpha_1) - (\alpha_1)] \dot{s}(\theta) \quad (6) \]

\[ (\phi_1) = (\phi_1) + [(\phi_1) - (\phi_1)] \dot{s}(\theta) \quad (7) \]

where \( s(\theta) = \sin(\theta/(4\pi)) \) for \( \theta > (2\pi \theta) \geq 1 \), for \( \theta/(2\pi \theta) \geq 1 \), and the control speed parameter \( \tau \) represents the number of orbits allowed for the maneuver.

Fig. 2 Typical system response showing pitch maneuver of lower satellite platform starting from \( \alpha_2 = 0 \) to 10 deg.

Fig. 3 Typical system response showing roll maneuver of lower satellite platform starting from \( \phi_2 = 0 \) to 15 deg.
Table 1  Minimum tether length requirements for various dual-satellite systems ($C = 3 \times 10^4$, $\alpha = 0.2$, and $\hat{b} = 1$)

<table>
<thead>
<tr>
<th>System data</th>
<th>Small satellites</th>
<th>Medium satellites</th>
<th>Large satellites</th>
<th>Small-medium satellites</th>
<th>Space shuttle</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1 - m_2$, kg</td>
<td>500–500</td>
<td>1000–1000</td>
<td>$10^3 - 10^5$</td>
<td>500–1000</td>
<td>$5 \times 10^3 - 10^5$</td>
</tr>
<tr>
<td>$I_x^1 - I_x^2$, kg m$^2$</td>
<td>100–100</td>
<td>500–500</td>
<td>$10^2 - 10^4$</td>
<td>100–500</td>
<td>$10^2 - 10^3$</td>
</tr>
<tr>
<td>$I_y^1 - I_y^2$, kg m$^2$</td>
<td>150–150</td>
<td>1000–1000</td>
<td>$10^3 - 10^4$</td>
<td>150–1000</td>
<td>$10^2 - 10^4$</td>
</tr>
<tr>
<td>$I_z^1 - I_z^2$, kg m$^2$</td>
<td>75–75</td>
<td>500–500</td>
<td>$8 \times 10^2 - 8 \times 10^4$</td>
<td>75–500</td>
<td>$8 \times 10^2 - 8 \times 10^6$</td>
</tr>
<tr>
<td>$L_{ref}$, m</td>
<td>0.45</td>
<td>0.71</td>
<td>10.0</td>
<td>1.00</td>
<td>14.2</td>
</tr>
<tr>
<td>$(\alpha_1)<em>{max}$ or $(\phi_1)</em>{max}$</td>
<td>1.2</td>
<td>1.8</td>
<td>25</td>
<td>2.5</td>
<td>29</td>
</tr>
</tbody>
</table>

\[
\text{Fig. 4} \text{ Typical chase-slewing maneuver response for desired harmonic variations in pitch angle for lower satellite platform.}
\]

\[
\begin{align*}
C &= 5 \times 10^4 K_x - 0.5, K_y = 0.3, I_{tot} = 15 \\
\delta_1 &= 0, \delta_2 = 0.2, b = 0.5, f = 1, g = 1 \\
\alpha' &= \alpha, \gamma_x = \gamma_y, I_z = 0 \\
\alpha_1 &= \alpha_1, \gamma_x = \gamma_y, I_z = 0 \\
\phi_1 &= \phi_1, \gamma_x = \gamma_y, I_z = 0 \\
\phi_2 &= \phi_2, \gamma_x = \gamma_y, I_z = 0 \\
\end{align*}
\]

Results and Discussion

With a view to assess the effectiveness for the proposed attitude maneuver scheme, the detailed system response is simulated through numerical integration of the exact system equations, namely Eqs. (1) and (2). For convenience two different classes of system models based on relative mass and inertia properties for the two satellite platforms considered for simulation are as follows.

1. Class 1: With identical satellite platforms $f = (m_1/m_2) = 1$, and $g = (I_x/I_z) = 1$.

2. Class 2: With nonidentical satellite platforms $f = (m_1/m_2) = 2$, and $g = (I_x/I_z) = 5$.

It is proposed to achieve the slewing attitude maneuver for satellite platform 2, while platform 1 is to be kept in the fixed orientation with its $y_1$ axis aligned with the local vertical. Figure 2 illustrates a typical maneuver from initial pitch angle, e.g., $\phi_1 = 0$ deg, to final pitch angle, e.g., $\phi_2 = 10$ deg. Figure 3 presents the corresponding case for roll maneuver from initial angle, e.g., $\psi_1 = 0$ deg, to final roll angle, e.g., $\psi_2 = 15$ deg. As expected, these maneuvers are accompanied by steady-state periodic $\beta, \eta$ oscillations. Besides, during the roll maneuver, the satellite yaw motion gets excited in view of its strong coupling with the roll degree of freedom. However, the attitude drifts remain low throughout the maneuver.

For chase-slewing maneuver we look at the specific situation of continual harmonic pitch or roll variations so as to follow a prespecified time history as described next:

\[
(\alpha_2)_i = (\alpha_1)_i + [(\alpha_2)_i - (\alpha_2)_i] \sin(\theta/(4\pi))
\]

\[
(\phi_2)_i = (\phi_1)_i + [(\phi_2)_i - (\phi_2)_i] \sin(\theta/(4\pi))
\]

\[
(\psi_2)_i = (\psi_1)_i + [(\psi_2)_i - (\psi_2)_i] \sin(\theta/(4\pi))
\]

Figure 4 demonstrates the effectiveness of the proposed open-loop tether length control for the pitch maneuver. Evidently, the larger are the changes in satellite orientation, the larger would be the associated attitude errors. Furthermore, of the pitch and roll maneuvers the latter one appears to involve relatively larger attitude errors. However, in all cases studied here, the attitude maneuver errors are found to remain well within small fraction of a degree.

It is now proposed to examine the effect of size of the two satellite platforms on actual tether lengths required. The data presented in Table 1 cover practically all possible satellite platform configurations comprising a light, medium, or large satellite tethered to similar satellites or with one having very different mass and inertia properties. In general, the tether length requirements increase with an increase in the size of the two satellites as expected. Short tethers, no longer than 3 m, are now adequate for small and even medium size satellites. With tethers being very short, it may now be feasible to provide for adequate protective shields around the tethers for safety against micrometeorite impacts and without excessive weight penalty.

Conclusions

The open-loop tether length control developed here is found to be quite effective for achieving desired general satellite slewing maneuver in pitch as well as roll degrees of freedom. In general, it is possible to limit the amplitudes of oscillations around desired final equilibrium position to rather small values. The excellent attitude maneuver characteristics are observed throughout, regardless of the class of system comprising two satellite platforms having identical or widely differing masses and inertia properties. That enables much greater flexibility in the choice of satellite mass distribution. The nearly passive nature of the proposed control mechanism using very short tethers makes the concept particularly attractive for future space missions.

References


Variable Attitude Compensation Through Tether for Comsats in Drifting/Inclined Geosynchronous Orbits

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Nomenclature

\( a_j \) = coordinate of attachment point \( j \); is 0 for \( j = 1, 2, \) and \((-1)/a \) for \( j = 3, 4 \)
\( C \) = \( E A/(mL_{ref}a_j^2) \); TSS rigidity parameter
\( c_j \) = coordinate of attachment point \( j \); is \((-1)/a \) for \( j = 1, 2 \) and 0 for \( j = 3, 4 \)
\( E A \) = tether modulus of rigidity, \( N \)
\( I_x, I_y, I_z \) = principal satellite moments of inertia about \( x, y, \) and \( z \) axes, respectively
\( i_b \) = orbit inclination, deg
\( j \) = tether points on satellite; 1, 2, 3, 4 for \( A, B, C, D \), respectively
\( K_p \) = inertia parameters; \( p = 1, 2, \ldots, 5 \);
\( K_1 = (l_i - l_4)/l_4, K_2 = (l_1 - l_4)/l_4, K_3 = (l_6 - l_1)/l_1, K_4 = 1 - K_1, K_5 = K_5 \)
\( L_1, L_0 \) = stretched and nominal unstretched lengths of \( j \)th tether, respectively
\( L_{ref} \) = reference length; \((L_1/m)^{1/2} \)
\( L_{10} \) = nominal unstretched lengths of four tethers
\( L_0 \) = \( L \) when tether strains are zero
\( l_i, l_j, l_{j0}, l_{j0} \) = \( L_i/L_{ref}, L_j/L_{ref}, L_{j0}/L_{ref} \), and \( L_{10}/L_{ref} \), respectively
\( U(\varepsilon_j) \) = 1 for \( \varepsilon_j \geq 0 \) and 0 for \( \varepsilon_j < 0 \)
\( \varepsilon_j \) = tether strains in the \( j \)th tether
\( \lambda_j \) = Lagrange multipliers
\( Q_{j0} \) = spin velocity of the Earth
\( r(S) \) = \(/L_{ref} \)
\( r(\cdot) \) = \( r(\cdot) \) for \( j \)th tether; \( j = 1, 2, 3, 4 \) for tethers
\( E - A, E - B, E - C, E - D \)
\( \theta(\cdot) \) = \( \theta(\cdot) \) at \( \theta \) (true anomaly) = 0
\( \| \cdot \|_{\max} \) = maximum amplitude of \( \cdot \)
\( \cdot, \cdot \) = \( \cdot, \cdot \) and \( \cdot, \cdot \) / \( \cdot, \cdot \), respectively

Introduction

The geostationary communications satellites undergo significant, continual changes in their orbital elements under the influence of environmental perturbations. Of these, the adverse secular effect on the orbital inclination is of considerable practical importance. It causes the satellites to undergo continually growing periodic lateral/longitudinal satellite drifts as viewed from the ground terminal. Rather expensive onboard fuel is periodically utilized for station-keeping maneuvers to ensure continual, uninterrupted communications. Here, it is proposed to explore the feasibility for developing a variable attitude controller using tethered auxiliary mass for the control of satellite tilting so as to effectively compensate for the periodic longitudinal and lateral drifts for satellites in 24-h nonequatorial circular orbits with respect to an equatorial ground station. This study is based on results of some earlier investigations \(^{1-7}\) that have established the effectiveness of the tethered satellite system (TSS) for satellite attitude stabilization and maneuver. In the light of the rapid worldwide growth in demands on communications capacity, extension of the satellite applications to new areas such as information technology and the associated problems of excessive overcrowding of the geostationary ring, \(^{8-16}\) this investigation may be of considerable significance.

Proposed Controller Model and Equations of Motion

This investigation considers a satellite moving in a nonequatorial, 24-h orbit. The satellite is assumed to be vertically above the ground station while passing over the nodes (Fig. 1). The line through the ascending node represents the reference line in orbit for measurement of the true anomaly \( \theta \). The coordinate frame \( x_0, y_0, z_0 \), passing through the system center of mass \( S \) with \( y_0 \) pointing along the local vertical and \( x_0 \) along the normal to the orbital plane, represents the local orbital reference frame. Three successive rotations of this local frame, \( \alpha \) (pitch), \( \gamma \) (yaw), and \( \phi \) (roll), lead to the general satellite orientation represented by its body frame \( S = x y z \).

The proposed satellite controller model is composed of an auxiliary mass deployed using four identical tethers attached to four distinct points on satellite surface (Fig. 1). The attachment points lie in a plane parallel to the satellite-yaw plane in a symmetric pattern with the \( x y z \) coordinates given as \( A = (0, -b, a), B = (0, -b, -a), C = (-a, -b, 0), \) and \( D = (a, -b, 0) \). Here \( a \) and \( b \) denote yaw plane and vertical offsets for satellite-tether attachment points.

The pendulum-like auxiliary mass \( m \), being much smaller than the satellite mass \( M \), is treated as a particle. Transverse vibrations of thin tethers, made of a light but rigid material like Kevlar and assumed to have negligible mass, are ignored. Similarly, for the variable vector length \( L \) joining the auxiliary mass to satellite mass center \( S \), two successive rotations, \( \beta \) about the \( x_0 \) axis referred to as

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Fig. 1 Geometry of motion of TSS in inclined, 24-h circular orbits relative to an equatorial ground station.