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Engineering Notes

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Pitch and Roll Attitude Maneuver of Twin-Satellite Systems Through Short Tethers

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Nomenclature

a_i	= yaw-plane tether offsets for points A_i, B_i, C_i, and D_i satellite i from mass-center S_i; $i = 1, 2$ (Fig. 1); $a_1 = 0$, $a_2 = a$, m
b_i	= common vertical offsets (y_i coordinates) of satellite points A_i, B_i, C_i, and D_i; $i = 1, 2$ (Fig. 1); $b = b_2 = -b_1$, m
C	= $EA/(m_2 L_{\mathrm{ref}} \Omega^2)$; TSS rigidity parameter, where Ω = Earth's spin rate
EA	= tether modulus of rigidity, N
f, g	= ratios of satellite masses (m_1/m_2), moments of inertia (I_{x_1}/I_{x_2})
$h_k(x, y, \ldots)$	= functions of $x, y, \ldots$; $k = 1, 2, \ldots, 9$
K_p	= inertia parameters, $p = 1, 2, 3$; $K_1 = (I_{x_i} - I_{y_i})/I_{z_i}$, $K_2 = (I_{y_i} - I_{z_i})/I_{x_i}$, $K_3 = (I_{x_i} - I_{z_i})/I_{y_i}$; same for $i = 1$ and 2
L_j, L_{j0}	= stretched and controlled unstretched length of jth tether, $L_j = L_{j0}(1 + \epsilon_j)$, m
L_{t0}, L_{ref}	= nominal lengths of four tethers, m and reference length given by $(I_{x_1}/m_2)^{1/2}$, m
L_0	= distance L when tether strains are zero, m
l, l_j, l_{j0}, l_{t0}	= L/L_{ref}, L_j/L_{ref}, L_{j0}/L_{ref}, L_{t0}/L_{ref}, respectively
m_i, I_{u_i}	= mass, kg, and principal moment of inertia for satellite i about u_i axis; $u = x, y, z$, kg m^2
q_1, q_2, q_3	= pitch(α_1), yaw(γ_1), and roll(ϕ_1), respectively, for satellite platform 1, deg
q_4, q_5, q_6	= pitch(α_2), yaw(γ_2), and roll(ϕ_2), respectively, for satellite platform 2, deg
$U(\epsilon_j)$	= 1 for $\epsilon_j \geq 0$ and 0 for $\epsilon_j < 0$
β, η	= relative in-plane (pitch) and out-of-plane (roll) swing angles of $\boldsymbol{L}$, deg
ϵ_j	= tether strains in the jth tether
θ	= true anomaly as measured from the reference line
$(\hat{\cdot})$	= $(\cdot)/L_{\mathrm{ref}}$
$(\cdot)_j$	= $(\cdot)$ for jth tether; $j = 1, 2, \ldots, 4$ for $A_1 - A_2$, $A_1 - B_2, \ldots, A_1 - D_2$, respectively
$(\cdot)_0$, $\lvert(\cdot)\rvert_{\max}$	= $(\cdot)$ at θ(true anomaly) = 0 and maximum amplitude of $(\cdot)$
$(\cdot)'$, $(\cdot)''$	= $\mathrm{d}(\cdot)/\mathrm{d}\theta$ and $\mathrm{d}^2(\cdot)/\mathrm{d}\theta^2$, respectively

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Introduction

SEVERAL investigators have proposed the use of tether(s) through active feedback control of their tension or offsets, alone or in combination.[1] More recently, the authors have established the feasibility of using a multitethered auxiliary mass to augment the stabilizing effect of gravity gradient moments and hence ensure the desired fixed satellite orientation.[2,3] Later, simple open-loop tether length control laws were developed to utilize the tether tensions for satellite attitude maneuver.[4–6] In a related subsequent development the authors proposed splitting of the spacecraft into two halves and suitably connecting them through tethers so that each of the two halves plays the role of an auxiliary mass stabilizing the other, and as a consequence a high degree of pointing stability is ensured for the whole system.[7]

Another related investigation[8] demonstrated how the tether length control in dual tethered satellite systems based on judiciously developed control laws could be made use of for a very specific satellite pitch maneuver. Here, it is proposed to generalize this investigation by exploring the feasibility for achieving fixed or continual chase slewing maneuver of lower satellite or platform in pitch and/or roll modes while the other one is held along the nominal local vertical. For this, the model proposed (Fig. 1) comprises two satellites connected through four identical tethers in an inverted parachutelike conical configuration.

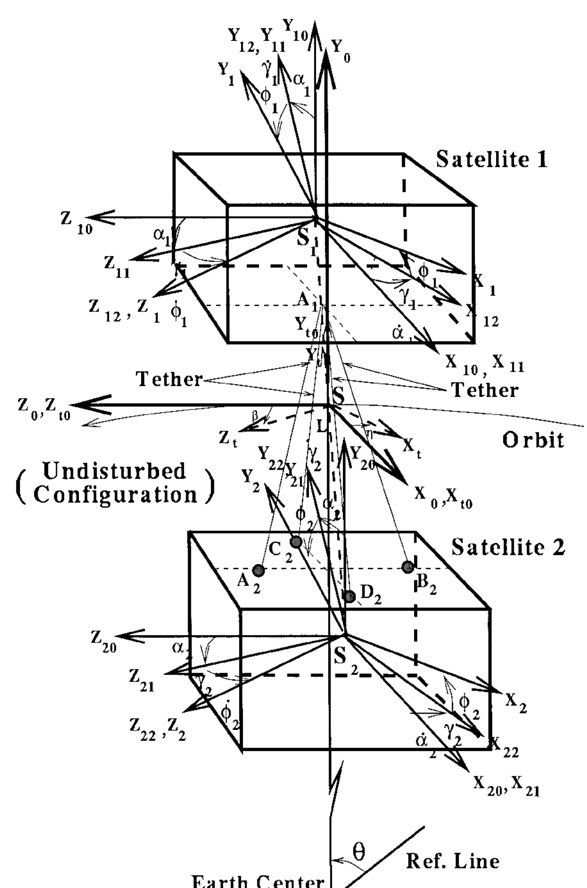

Fig. 1 Geometry of dual-tethered satellite system.

Equations of Motion

The Lagrangian formulation approach with Lagrange multipliers has been adopted to obtain the governing equations of motion for the constrained system. For brevity these equations taken from Ref. 9 are presented in concise symbolic form as follows:

$$h_k(q_p'', q_p', q_p, \beta, \eta, l, K_1, K_3) - gC\sum_{j=1}^{4}[\epsilon_j U(\epsilon_j)]\frac{\partial f_j}{\partial q_k} = 0$$

$$(k = 1, 2, \ldots, 6;\ p = 1, 2, \ldots, 6)$$

$$h_7(\beta'', \beta', \beta, \eta', \eta, l, l') - \left(1 + \frac{1}{f}\right)\left(\frac{1}{l\cos\eta}\right)^2 C\sum_{j=1}^{4}[\epsilon_j U(\epsilon_j)]\frac{\partial f_j}{\partial \beta} = 0$$

$$h_8(\eta'', \eta', \eta, \beta', \beta, l, l') - \left(1 + \frac{1}{f}\right)(1/l^2)C\sum_{j=1}^{4}[\epsilon_j U(\epsilon_j)]\frac{\partial f_j}{\partial \eta} = 0$$

$$h_9(l'', l, \beta', \beta, \eta', \eta) - \left(1 + \frac{1}{f}\right)C\sum_{j=1}^{4}[\epsilon_j U(\epsilon_j)]\frac{\partial f_j}{\partial l} = 0 \tag{1}$$

subject to the constraint relations

$$f_j = f_j(\hat{a}_i, \hat{b}_i, q_k, \beta, \eta, l, l_j) = 0$$

$$(j = 1, 2, 3, 4;\ i = 1, 2;\ k = 1, 2, \ldots, 6) \tag{2}$$

Synthesis of Open-Loop Control Laws

The governing complex nonlinear set of equations of motion is first examined for its feasible equilibrium configurations and their likely dependence on system parameters. For the twin-satellite system positioned in a fixed arbitrary steady-state configuration given by, e.g., α_i, ϕ_i, γ_i, β, η, l, a substitution of their equilibrium values and zero for their first and second derivatives into Eqs. (1) and (2) and carrying out a series of steps as outlined in Ref. 9, we can show that for achieving the desired orientation the tether lengths must be regulated according to the relations

$$l_{j0} = l_{t0} - (-1)^j\hat{a}[l_0\cos\eta\sin(\alpha_2 - \beta) - \hat{b}_1\cos\phi_1\sin(\alpha_2 - \alpha_1)]/l_{t0}, \qquad j = 1, 2$$

$$l_{j0} = l_{t0} - (-1)^j\hat{a}\{l_0[\sin\phi_2\cos\eta\cos(\alpha_2 - \beta) - \sin\eta\cos\phi_2] - \hat{b}_1\sin\phi_2\cos\alpha_2\}/l_{t0}, \qquad j = 3, 4 \tag{3}$$

where

$$\beta = \tfrac{1}{2}\sin^{-1}\left\{(1 + 1/f)(1/l^2) \times (K_1 - K_3)\sin 2\alpha_2/[g(K_1K_3 - 1)]\right\} \tag{4}$$

$$\eta = \tfrac{1}{2}\sin^{-1}\left\{(1 + 1/f)(1/l^2)K_1 \times (1 - K_3)\sin 2\phi_2/[g(K_1K_3 - 1)]\right\} \tag{5}$$

For preliminary design estimates of tether lengths needed, the linearized perturbation pitching stabilty analysis was undertaken. The study was facilitated through sacrifice of rigor by way of several simplifying assumptions, e.g., $g = 1$, and $K_2 = 1$ corresponding to the unstable gravity gradient platform configurations. The stability criterion thus obtained can be written as

$$L_{t0} > (1 + 1/f)L_{\mathrm{ref}}^2\big|\ b$$

Even during stable attitude maneuvers, the transient disturbances are inherently present. It is easy to see that these can be minimized if the law regulating the equilibrium pitch or roll orientations is chosen as follows:

$$(\alpha_2)_f = (\alpha_2)_i + [(\alpha_2)_f - (\alpha_2)_i]s(\theta) \tag{6}$$

$$(\phi_2)_f = (\phi_2)_i + [(\phi_2)_f - (\phi_2)_i]s(\theta) \tag{7}$$

where $s(\theta) = \sin[\theta/(4\tau)]$ for $1 > \theta/(2\pi\tau) \geq 0$, 1 for $\theta/(2\pi\tau) \geq 1$, and the control speed parameter τ represents the number of orbits allowed for the maneuver.

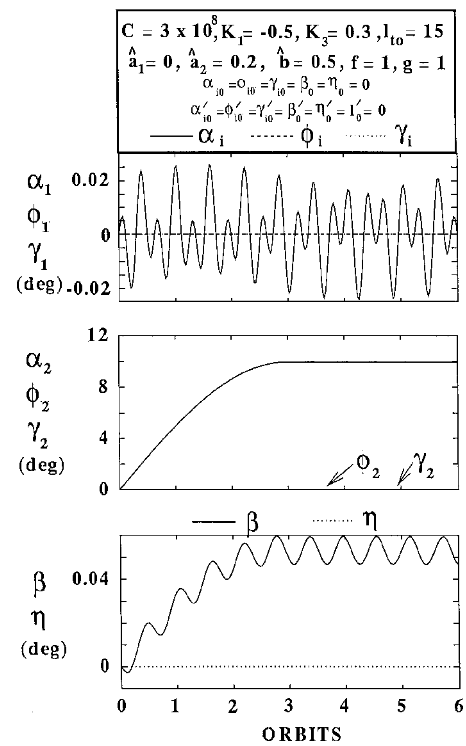

Fig. 2 Typical system response showing pitch maneuver of lower satellite platform starting from $\alpha_2 = 0$ to 10 deg.

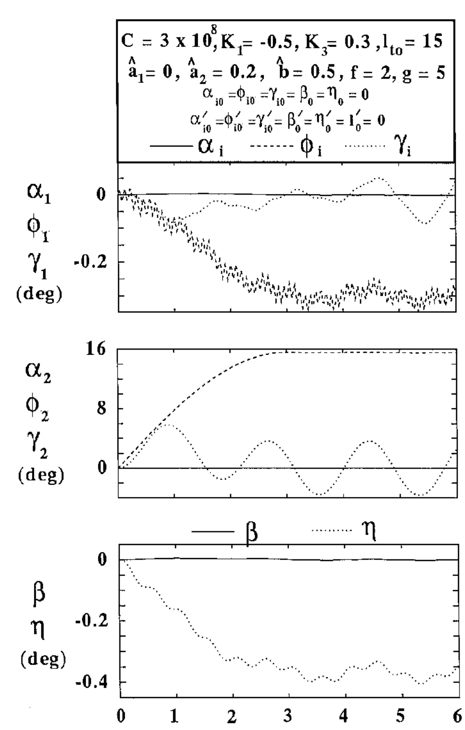

Fig. 3 Typical system response showing roll maneuver of lower satellite platform starting from $\phi_2 = 0$ to 15 deg.

Table 1 Minimum tether length requirements for various dual-satellite systems ($C = 3 \times 10^8$, $\hat{a} = 0.2$, and $\hat{b} = 1$)

System	Class 1			Class 2	
data	Small satellites	Medium satellites	Large satellites	Small–medium satellites	Space shuttle
$m_1 - m_2$, kg	500–500	1000–1000	$10^5 - 10^5$	500–1000	$5 \times 10^4 - 10^5$
$I_{x_1} - I_{x_2}$, kg m^2	100–100	500–500	$10^7 - 10^6$	100–500	$10^6 - 10^7$
$I_{y_1} - I_{y_2}$, kg m^2	150–150	1000–1000	$10^7 - 10^7$	150–1000	$10^6 - 10^7$
$I_{z_1} - I_{z_2}$, kg m^2	75–75	500–500	$8 \times 10^6 - 8 \times 10^6$	75–500	$8 \times 10^5 - 8 \times 10^6$
L_{ref}, m	0.45	0.71	10.0	1.00	14.2
($\lvert\alpha_i\rvert_{\max}$ or $\lvert\phi_i\rvert_{\max}$ = 30 deg) $(L_{t0})_{\min}$, m	1.2	1.8	25	2.5	29

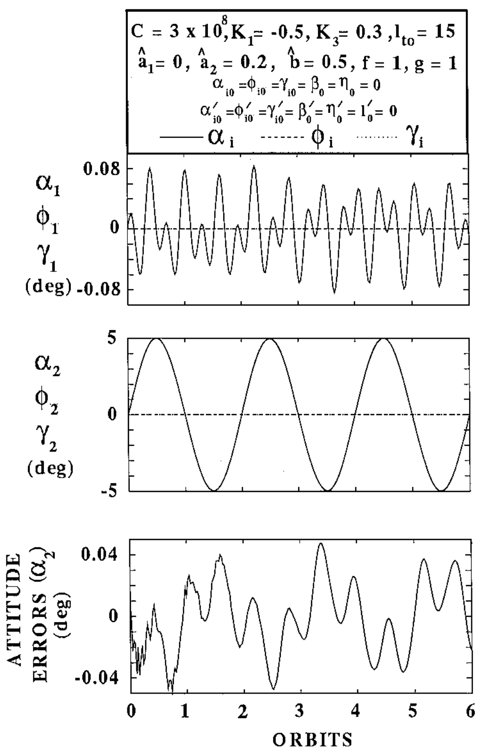

Fig. 4 Typical chase-slewing maneuver response for desired harmonic variations in pitch angle for lower satellite platform.

Results and Discussion

With a view to assess the effectiveness for the proposed attitude maneuver scheme, the detailed system response is simulated through numerical integration of the exact system equations, namely Eqs. (1) and (2). For convenience two different classes of system models based on relative mass and inertia properties for the two satellite platforms considered for simulation are as follows.

1) Class 1: With identical satellite platforms $f = (m_1/m_2) = 1$, and $g = (I_{x_1}/I_{x_2}) = 1$.

2) Class 2: With nonidentical satellite platforms $f = (m_1/m_2) = 2$, and $g = (I_{x_1}/I_{x_2}) = 5$.

It is proposed to achieve the slewing attitude maneuver for satellite platform 2, while platform 1 is to be kept in the fixed orientation with its y_1 axis aligned with the local vertical. Figure 2 illustrates a typical maneuver from initial pitch angle, e.g., $\alpha_2 = 0$ deg, to final pitch angle, e.g., $\alpha_2 = 10$ deg. Figure 3 presents the corresponding case for roll maneuver from initial angle, e.g., $\phi_2 = 0$ deg, to final roll angle, e.g., $\phi_2 = 15$ deg. As expected, these maneuvers are accompanied by steady-state periodic β, η oscillations. Besides, during the roll maneuver, the satellite yaw motion gets excited in view of its strong coupling with the roll degree of freedom. However, the attitude drifts remain low throughout the maneuver.

For chase-slewing maneuver we look at the specific situation of continual harmonic pitch or roll variations so as to follow a prespecified time history as described next:

$$(\alpha_2)_f = (\alpha_2)_i + [(\alpha_2)_f - (\alpha_2)_i]\sin[\theta/(4\tau)] \tag{8}$$

$$(\phi_2)_f = (\phi_2)_i + [(\phi_2)_f - (\phi_2)_i]\sin[\theta/(4\tau)] \tag{9}$$

Figure 4 demonstrates the effectiveness of the proposed open-loop tether length control for the pitch maneuver. Evidently, the larger are the changes in satellite orientation, the larger would be the associated attitude errors. Furthermore, of the pitch and roll maneuvers the latter one appears to involve relatively larger attitude errors. However, in all cases studied here, the attitude maneuver errors are found to remain well within small fraction of a degree.

It is now proposed to examine the effect of size of the two satellite platforms on actual tether lengths required. The data presented in Table 1 cover practically all possible satellite platform configurations comprising a light, medium, or large satellite tethered to similar satellites or with one having very different mass and inertia properties. In general, the tether length requirements increase with an increase in the size of the two satellites as expected. Short tethers, no longer than 3 m, are now adequate for small and even medium size satellites. With tethers being very short, it may now be feasible to provide for adequate protective shields around the tethers for safety against micrometeorite impacts and without excessive weight penalty.

Conclusions

The open-loop tether length control developed here is found to be quite effective for achieving desired general satellite slewing maneuver in pitch as well as roll degrees of freedom. In general, it is possible to limit the amplitudes of oscillations around desired final equilibrium position to rather small values. The excellent attitude maneuver characteristics are observed throughout, regardless of the class of system comprising two satellite platforms having identical or widely differing masses and inertia properties. That enables much greater flexibility in the choice of satellite mass distribution. The nearly passive nature of the proposed control mechanism using very short tethers makes the concept particularly attractive for future space missions.

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Variable Attitude Compensation Through Tether for Comsats in Drifting/Inclined Geosynchronous Orbits

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Nomenclature

a_j	= x coordinate of attachment point j; is 0 for $j = 1, 2$ and $(-1)^j a$ for $j = 3, 4$
C	= $EA/(mL_{\mathrm{ref}}\Omega_{\mathrm{ea}}^2)$; TSS rigidity parameter
c_j	= z coordinate of attachment point j; is $(-1)^j a$ for $j = 1, 2$ and 0 for $j = 3, 4$
EA	= tether modulus of rigidity, N
I_x, I_y, I_z	= principal satellite moments of inertia about x, y, and z axes, respectively
i_b	= orbit inclination, deg
j	= tether points on satellite; 1, 2, 3, 4 for A, B, C, D, respectively
K_p	= inertia parameters: $p = 1, 2, \ldots, 5$; $K_1 = (I_x - I_y)/I_z$, $K_2 = (I_y - I_z)/I_x$, $K_3 = (I_x - I_z)/I_y$, $K_4 = 1 - K_1K_5$, $K_5 = (K_3 - 1)/(K_1K_3 - 1)$
L_j, L_{j0}	= stretched and nominal unstretched length of jth tether, respectively
L_{ref}	= reference length; $(I_x/m)^{1/2}$
L_{t0}	= nominal unstretched lengths of four tethers
L_0	= L when tether strains are zero
l, l_j, l_{j0}, l_{t0}	= L/L_{ref}, L_j/L_{ref}, L_{j0}/L_{ref}, and L_{t0}/L_{ref}, respectively
$U(\epsilon_j)$	= 1 for $\epsilon_j \geq 0$ and 0 for $\epsilon_j < 0$
ϵ_j	= tether strains in the jth tether
λ_j	= Lagrange multipliers
Ω_{ea}	= spin velocity of the Earth
$\hat{()}$	= $(.)/L_{\mathrm{ref}}$
$(.)_j$	= (.) for jth tether; $j = 1, 2, 3, 4$ for tethers $E-A$, $E-B$, $E-C$, $E-D$
$(.)_0$	= (.) at θ(true anomaly) $= 0$
$\lvert (.) \rvert_{\max}$	= maximum amplitude of (.)
$(.)'$, $(.)''$	= $\mathrm{d}(.)/\mathrm{d}\theta$ and $\mathrm{d}^2(.)/\mathrm{d}\theta^2$, respectively

Introduction

THE geostationary communications satellites undergo significant continual changes in their orbital elements under the influence of environmental perturbations. Of these, the adverse secular effect on the orbital inclination is of considerable practical importance. It causes the satellites to undergo continually growing periodic lateral/longitudinal satellite drifts as viewed from the ground terminal. Rather expensive onboard fuel is periodically utilized for station-keeping maneuvers to ensure continual, uninterrupted communications. Here, it is proposed to explore the feasibility for developing a variable attitude controller using tethered auxiliary mass for continual satellite tilting so as to effectively compensate for the periodic longitudinal and lateral drifts for satellites in 24-h nonequatorial circular orbits with respect to an equatorial ground station. This study is based on results of some earlier investigations[1–7] that have established the effectiveness of the tethered satellite system (TSS) for satellite attitude stabilization and maneuver. In the light of the rapid worldwide growth in demands on communications capacity, extension of the satellite applications to new areas such as information technology and the associated problems of excessive overcrowding of the geostationary ring,[8–10] this investigation may be of considerable significance.

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Proposed Controller Model and Equations of Motion

This investigation considers a satellite moving in a nonequatorial, 24-h orbit. The satellite is assumed to be vertically above the ground station while passing over the nodes (Fig. 1). The line through the ascending node represents the reference line in orbit for measurement of the true anomaly θ. The coordinate frame x_0, y_0, z_0, passing through the system center of mass S with y_0 pointing along the local vertical and x_0 along the normal to the orbital plane, represents the local orbital reference frame. Three successive rotations of this local frame, α (pitch), γ (yaw), and ϕ (roll), lead to the general satellite orientation represented by its body frame $S - xyz$.

The proposed satellite controller model is composed of an auxiliary mass deployed using four identical tethers attached to four distinct points on satellite surface (Fig. 1). The attachment points lie in a plane parallel to the satellite-yaw plane in a symmetric pattern with the xyz coordinates given as $A \equiv (0, -b, a)$, $B \equiv (0, -b, -a)$, $C \equiv (-a, -b, 0)$, and $D \equiv (a, -b, 0)$. Here a and b denote yaw plane and vertical offsets for satellite-tether attachment points.

The pendulumlike auxiliary mass m, being much smaller than the satellite mass M, is treated as a particle. Transverse vibrations of thin tethers, made of a light but rigid material like Kevlar and assumed to have negligible mass, are ignored. Similarly, for the variable vector length $\boldsymbol{L}$ joining the auxiliary mass to satellite mass center S, two successive rotations, β about the x_0 axis referred to as

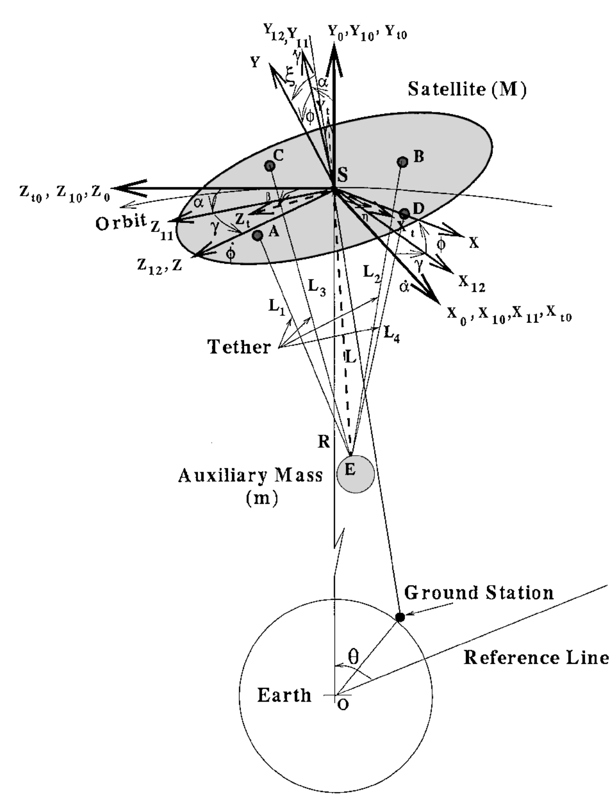

Fig. 1 Geometry of motion of TSS in inclined, 24-h circular orbits relative to an equatorial ground station.