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Performance of Bare-Tether Systems Under Varying Magnetic and Plasma Conditions

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Electrodynamic tethered systems, in which an exposed portion of the conducting tether itself collects electrons from the ionosphere, promise to attain currents of 10 A or more in low Earth orbit. For the first time, another desirable feature of such bare-tether systems is reported and analyzed in detail: Collection by a bare tether is relatively insensitive to variations in electron density that are regularly encountered on each revolution of an orbit. This self-adjusting property of bare-tether systems occurs because the electron-collecting area on the tether is not fixed, but extends along its positively biased portion, and because the current varies as collecting length to a power greater than unity. How this adjustment to density variations follows from the basic collection law of thin cylinders is shown. The effect of variations in the motionally induced tether voltage is also analyzed. Both power and thruster modes are considered. The performance of bare-tether systems to tethered systems is compared using passive spherical collectors of fixed area, taking into consideration recent experimental results. Calculations taking into account motional voltage and plasma density around a realistic orbit for bare-tether systems suitable for space station applications are also presented.

Nomenclature

$\boldsymbol{B}$ = Earth's magnetic field vector
C_{OML} = constant factor in orbital-motion-limited collection formula, $p/\pi\ \sqrt{(2e^3)/m_e}$
E = motional electric field component parallel to tether
e = electron charge
I = current collected from plasma and delivered to platform
L = tether length
L_C = electron-collecting length
L_{ins} = length of insulation on tether
$\hat{\ell}$ = unit vector pointing along tether
m_e = electron mass
N_e = unperturbed electron density
P = electrical power supplied for thruster
p = perimeter of tether
$V(y)$ = variable bias voltage of tether with respect to plasma
V_{end} = bias voltage at end of tether insulation (thruster)
V_{in} = input voltage applied to tether thruster
V_{tip} = bias voltage at upper end of tether (power generator)
$\boldsymbol{v}$ = velocity of tethered system with respect to plasma
Z = impedance of useful load in tether generator circuit
$\varepsilon_{\mathrm{EO}}$ = efficiency of electrical to orbital energy conversion (thruster)
$\varepsilon_{\mathrm{OE}}$ = efficiency of orbital energy to useful electrical energy conversion

Introduction

THE use of an uninsulated metallic wire to serve as the anode for electrodynamic tethered systems (the anode being part of the tether itself) was proposed some time ago,[1] and the concept will soon be tested in space when the Propulsive Small Expendable Deployer System (ProSEDS) mission is launched in the fall of 2000.[2] The greater efficiency of current collection by the bare tether compared with standard spherical anodes has been emphasized. Another attractive feature of the bare-tether anode, which has not been previously discussed in the literature, is the ability of a system based on the bare tether to adjust automatically to variations in electron density in a way that smoothes out variations in current as the system passes from daytime to nighttime operation along its orbit.

We first review the argument for bare tethers as more efficient electron collectors in the light of the Tethered Satellite System 1 Reflight (TSS-1R) Space Shuttle experiment results.[3] We then show in mathematical detail how the self-adjusting property of bare tethered systems arises naturally from the physics of thin wire current collection. We consider power generation and thruster modes separately, comparing results from a bare-tether system to spherical collector systems that are equivalent to the bare-tether system in a defined sense. We also show how the current collected in each mode varies with the motional voltage. For both power and thruster modes, we present results of numerical calculations for application of bare-tether systems to the International Space Station (ISS) in which all factors, including tether resistance and typical variations in plasma density and magnetic field vector, are taken into account. We summarize our conclusions in the final section.

Bare Tethers in Context

Electrodynamic tethers (EDTs) have been demonstrated to work in space, most notably by the TSS-1/R missions[3] and the plasma motor/generator (PMG) experiment.[4] In each case, a long conductive tether, covered by an insulating sheath, served as a conductive path for electrons at one end of the system to a higher electrical potential at the other end. Exchange of electrical charge with the ionosphere occurred at the ends of the system. The positively biased subsatellite served as the electron collector for TSS-1/R. At the Shuttle end, electrons were ejected by electron guns, and positive ions were collected by metallic surfaces. PMG used hollow cathodes for charge exchange at each end of the system, which operated in both the motionally biased (generator) mode and in a battery-imposed reversed bias (motor) mode. The current collected by TSS-1R exceeded expectations, most notably in the unscheduled

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1-A current collected immediately before and for about a minute after the tether break. The generally accepted explanation for the 1-A tether current flow after the tether break occurred is that ablated tether material and rapidly expelled air, which had been held within the tether before the break, ionized sufficiently to serve as a plasma contactor to expel electrons into the ionosphere.[5]

Whereas the experiments just mentioned demonstrate that the basic concept of EDTs is sound, they do not in themselves provide much encouragement for applications such as power generation or reboost for the ISS, which would require currents on the order of 10 A. A current of 0.3 A was attained by PMG, but the experiment failed to demonstrate efficient electron collection by a hollow cathode in space, particularly under low-density conditions. Before TSS-1R, the general assumption had been that current would be limited by the effect of magnetic guiding of electrons in accordance with the static Parker–Murphy limit,[6] which had been found to apply in earlier rocket-borne current collection experiments. The actual current collected by TSS-1R generally exceeded Parker–Murphy model predictions by a factor between two and three, but it still showed the square-root dependence on bias voltage predicted by the model, which was taken as strong evidence by some TSS-1R investigators that the Earth's magnetic field was the limiting factor in TSS-1R current collection.[7] By assuming that the observed square-root dependence continues to hold for higher voltages, the 1-A current collected by TSS-1R at a bias of around 1.5 kV (motional voltage was around 3.5 kV) implies that for the TSS-1 satellite to collect a current of 10 A would require a bias voltage of 150 kV. Under the same orbital conditions as held during the peak TSS-1R current flow, this would correspond to a tether length of 850 km, even neglecting the tether resistance.

Of course, increasing the bias voltage is not the only way to collect more current. A spherical surface larger than the roughly 8 m^2 of the TSS-1 satellite could be used to collect higher currents than TSS-1R attained. Unfortunately, increasing the area of the sphere almost certainly decreases the benefit of the bias voltage. The TSS-1R results may reflect Parker–Murphy current collected under conditions of enhanced thermal current[8] or an as yet undetermined mechanism for overcoming the magnetic insulation. The question is not likely to be resolved until there are further experimental results. However, the current collected per unit area by a sphere (under fixed conditions) decreases with the sphere's area in both models put forward to describe the TSS-1R current collection results.[8,9] This natural law of diminishing returns is shown in Fig. 1. A fixed bias voltage of 1.5 kV, electron density of $1.0 \times 10^{12}/m^3$, electron temperature of 0.16 eV, and geomagnetic field strength of 0.3 G have been assumed in Fig. 1, which shows the current collected as a function of total surface area in the range 8–400 m^2. The straight line shows the result of naively scaling the current up with the collecting area. The two curves correspond to calculations based on different collection models. One curve (labeled 2 $\times$PM) assumes that current collected by the sphere is twice the Parker–Murphy limit (in rough agreement with TSS-1R). In the Parker–Murphy model, due to the increasing importance of magnetic effects (binding of electrons to field lines) as the dimensions of the sphere greatly exceed the electron gyroradius, the current collected per unit area approaches half the thermal current density for large radii. The bias voltage provides no benefit for large radii. The other curve (labeled SC) is based on a space-charge-limited collection model[10] that ignores magnetic effects altogether. It has been claimed that this space-charge-limited model better fits the TSS-1R data,[9] though no theoretical motivation has been given for its application. In any case, as the sphere's radius increases, the ratio of the sphere's radius to the plasma sheath radius approaches unity, and the benefit of the bias voltage declines in the space-charge-limited model as well. The similarity of the two curves is remarkable. We use the Parker–Murphy model with the multiplicative factor of two in the calculations of collection by a sphere that follow. As we shall see, the spherical collector suffers from strong sensitivity to plasma density variations, so that a system that might collect sufficient current during peak daytime conditions would be greatly degraded during eclipse.

The so-called orbital-motion-limited (OML) current is the upper limit to current that can be collected by spherical and cylindrical probes in a collisionless plasma.[11] Small probes (spherical or cylindrical) with diameters less than the Debye length and electron gyroradius (both on the order of a centimeter in the ionosphere), by avoiding the effects of space charge shielding and magnetic guiding, collect OML current and are many times more efficient at collecting current (per surface area) than large balls such as the TSS collector. A sphere of such small dimensions is much too small to collect multiamp currents with a reasonable bias voltage, however. Because the area of a sphere is determined by its radius, the sphere's collecting surface cannot be made large without violating the required smallness conditions on its radius.

Because a thin cylinder (wire) remains a thin cylinder, no matter how long it is, the current collection problem for a wire is basically two dimensional. A thin wire should be a very efficient collector of current from the ionosphere, and its collecting area can be made large by increasing the length (kilometers), while keeping the diameter small enough to satisfy the conditions of OML current collection. That, in essence, is the bare-tether idea.

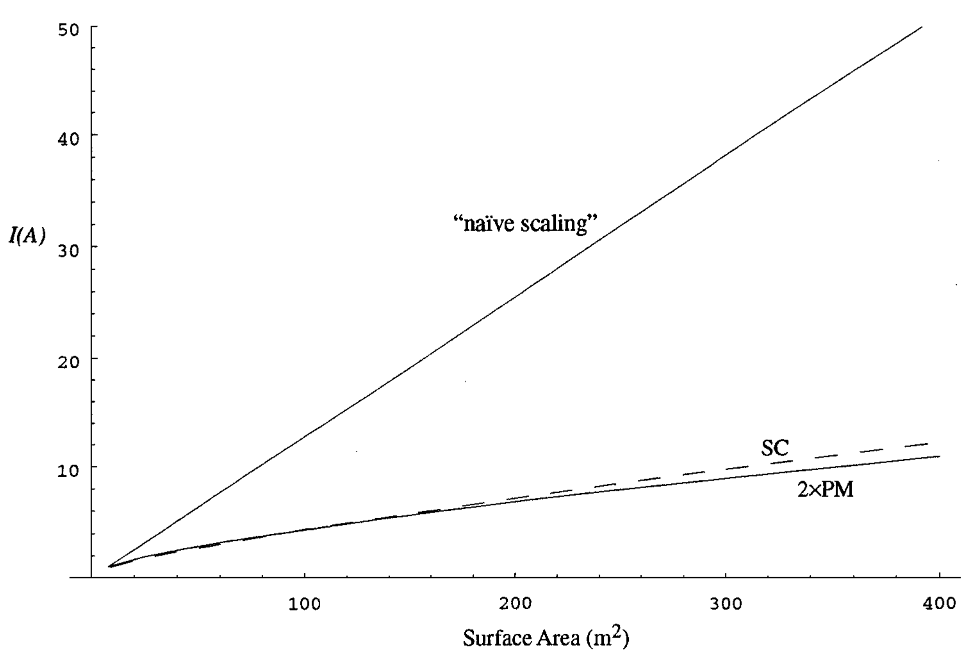


Fig. 1 Current collected vs collecting area for a passive sphere in typical daytime ionosphere, assuming twice Parker–Murphy (2 $\times$ PM) or space-charge-limited (SC) model.

To derive the general behavior of a bare-tether system under variations of plasma density and motional electric field, we make the simplifying assumption of negligible tether resistance in the following analysis. This enables us to make the results analytically transparent. No serious compromise is involved in this assumption because any useful tethered system would need to be efficient, which depends on the tether resistance being sufficiently low that the ohmic drop in the tether is much less than the motional voltage and, in the case of the thruster, the applied voltage.

Our analysis shows that, due to the variable collecting area of a bare tether and the form of the collection law, current collection is much less sensitive to variations in plasma density than is passive collection by a system based on a large (fixed area) spherical collector. We will demonstrate how this comes about for both the power generator and thruster modes of EDT operation. We wish to emphasize that this result depends on changes in the collecting length and in the bias voltage distribution along the tether, which result from the tether's being but one part of an electrical system that includes the ionosphere, other loads, and/or power sources. The current collected by a small length of tether is always directly proportional to the plasma density, as is the current collected by a unit surface of spherical conductor.

We also show that the efficiency of energy conversion is relatively steady under variations in motional electric field in both modes, but with different consequences for steadiness of the desired product in the two cases inasmuch as the motional electric field is the energy source for the EDT generator.

For both the power generator and thruster modes of operation, we proceed in the following way. First, we derive the equations for current collected by the bare-tether system in the limit of zero tether resistance. Then, to obtain an analytical expression for how the current varies as a function of plasma density and motional emf, we consider the limit of high-efficiency operation. In addition to being the regime of potentially greatest interest, the high-efficiency regime is where the bare-tether system exhibits markedly less sensitivity to plasma density variations than a comparable spherical-collector-based system. The analytical expression we obtain for the current enables us to see the physical roots of this desirable feature. Then we present numerical results of current collection by a bare-tether system, ignoring tether resistance but allowing density drops that take the system out of the regime of high-efficiency operation. Finally, we take a look at the performance of a less than optimal bare-tether system with realistic tether parameters, including nonzero tether resistance, in a realistic low Earth orbit. These last results are numerical calculations based on the development found in Ref. 1. In all of the following analysis we assume an eastward moving tethered system in low Earth orbit. The discussion of the power generator mode includes the basics of OML current collection, which is essential for the analysis of both modes.

Power Generator Mode

By power generator mode, we mean the mode in which electrons are collected from the ionosphere at the upper end of the system by virtue of the system's motional voltage. A system operating in the power generator mode is shown in Fig. 2. At the upper end of the system, the tether itself, which has been left exposed to the ionosphere, serves as the electron collector. Electrons travel down the tether and are expelled back into the ionosphere at the lower end by a plasma contactor that maintains the deployment platform at a low bias with respect to the ambient plasma.

The ProSEDS mission,[2] which is scheduled for a flight in the fall of 2000 as a Delta-II secondary payload to provide the first test in space of the bare-tether concept, will operate in the general configuration shown in Fig. 2. ProSEDS will in fact utilize electrical power from the tether to recharge its batteries and keep instruments, transmitters, and hollow cathode all working for the duration of the experiment, extending the useful lifetime of the system by days or weeks. Power generation is, however, only a secondary objective of the ProSEDS mission, and the system has not been optimized for this purpose. In addition to demonstrating bare-tether current collection in space, ProSEDS aims to demonstrate the feasibility for spacecraft propulsion of using the magnetic force exerted on an

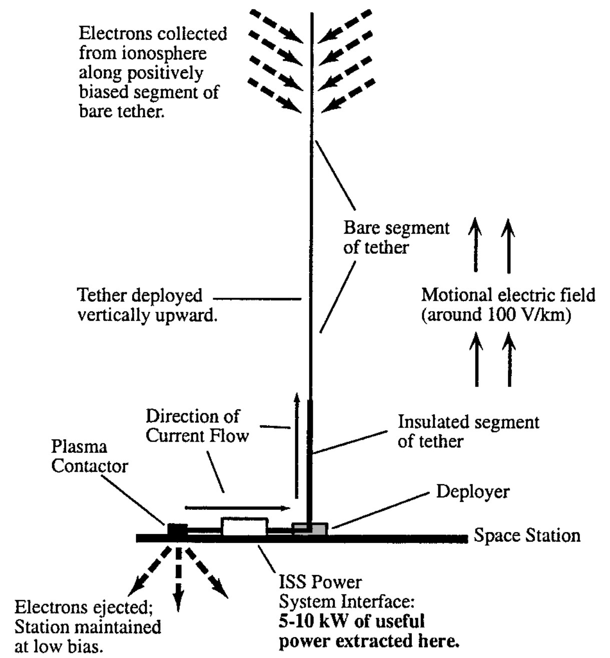


Fig. 2 Schematic diagram of a bare-tether power generator for the ISS.

EDT by the Earth's magnetic field. In the power generator mode, the current flow is in the direction to make this force a drag force, which is an inherent drawback of EDT power generation, because orbital energy is the source of the electrical energy. However, this drag can be a good thing, if accelerated re-entry is the goal. The force of the magnetic field on the current-carrying ProSEDS tether will also demonstrate the potential for a tether thruster in which the current flows in the opposite direction.

In the analysis that follows, we assume that we are dealing with a system designed to generate electrical power efficiently. One obvious requirement of efficiency is that the impedance of the useful load be much greater than the tether resistance, so that most of the orbital energy converted by the system is put to use rather than wasted in heating the tether. The basic operation of a bare-tether power generator can be deduced from the voltage diagram in Fig. 3, in which the vertical axis displays voltages (in the rest frame of the tethered system), and the horizontal axis represents distance along the upwardly deployed tether. At the lower end of the tether (Fig. 3, far right) an ideal hollow cathode maintains the deployment platform at the local plasma potential. The plasma potential decreases linearly with distance up the tether (moving to the left in Fig. 3), reaching a maximum negative value of EL at the upper tip of the tether, where we assume that the tether is straight.

We ignore the ohmic voltage drop in the tether because it is not the essential feature and would be a secondary effect for an efficiently designed power generation system. We assume that a useful load that utilizes the tether current and has an impedance Z is placed in series with the tether. The tether is then at a potential $-IZ$ with respect to the lower platform. Thus, the tether is negatively biased with respect to the local plasma as we move up the tether until we reach a point of zero bias. From there on out to the upper tip, the tether is positively biased. It is along this positively biased segment, designated L_C in Fig. 3, that the electron collection occurs. An ion current, which is much smaller than the electron current due the much greater mass of the ions, is collected along the negatively biased portion and slightly reduces the current to the useful load. We neglect the ion collection in this analysis.

We now concentrate on the positively biased segment, to calculate the electron current collected by the wire. In Fig. 4, the local bias of the tether with respect to the plasma is indicated by $V(y)$. The maximum bias, which occurs at the upper tip, is given by $V_{tip} = EL - IZ$. We assume the cross-sectional dimensions of the tether are sufficiently small that electrons are collected in the OML regime.[1] Generally speaking this means, for a tether of circular cross

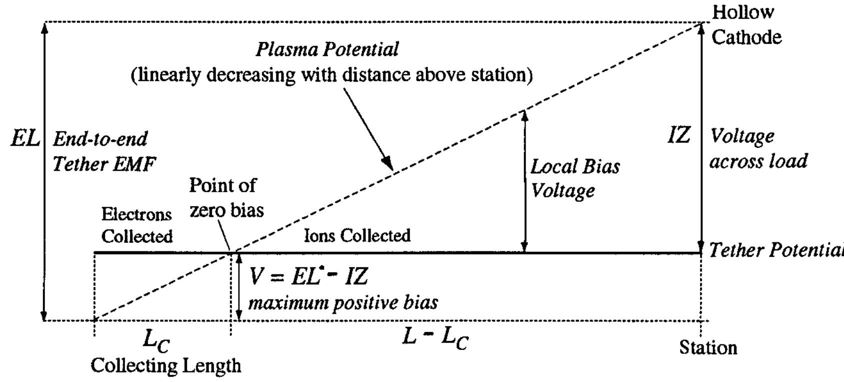


Fig. 3 Voltage diagram for an ideal bare-tether power generator as seen in tether rest frame.

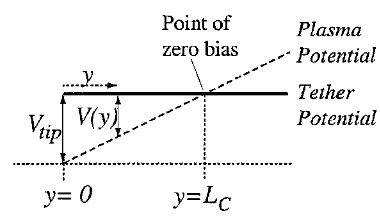


Fig. 4 Voltage diagram for bare-tether generator in region of positive tether bias.

section, a diameter smaller than both the Debye length and the electron gyroradius, but near-OML collection is attained for even larger dimensions.[12] In the daytime ionosphere encountered in low Earth orbit, these characteristic lengths are of the order of a centimeter.

When we consider only the dominant OML term, the fundamental equation for current collection by a wire along which the bias varies may be written as

$$\frac{\mathrm{d}I(y)}{\mathrm{d}y} = C_{\mathrm{OML}} N_e \sqrt{V(y)} \tag{1}$$

where $I(y)$ is the current flowing in the tether at position y in the positively biased section, the constant C_{OML} depends on the perimeter of the tether and the electron mass and charge,[1,11] and N_e is assumed constant along the tether. Integrating Eq. (1) to get the total electron current collected, we obtain

$$I = \int_0^{L_C} \frac{\mathrm{d}I}{\mathrm{d}y}\mathrm{d}y = \frac{2}{3}\frac{C_{\mathrm{OML}}}{E} N_e V_{\mathrm{tip}}^{\frac{3}{2}} = \frac{2}{3} C_{\mathrm{OML}} N_e \sqrt{E} L_C^{\frac{3}{2}} \tag{2}$$

Thus, the current collected is proportional to the $\frac{3}{2}$ power of the tip bias or, equivalently, the $\frac{3}{2}$ power of the collecting length. This $\frac{3}{2}$ power dependence has important consequences that give the bare tether a significant edge over passive spherical collectors, in terms of dependence on plasma density, as we shall see. The primary advantage comes from the size of the factor C_{OML}, however. This can be seen dramatically by comparing a bare tether collector to two spherical collectors, which we can term equivalent to it by different criteria. We ignore the tether resistance in considering these ideal systems.

For the bare-tether system, we take a 10-km-long wire with circular cross section and radius of 3.6 mm (well within the OML collecting regime). It generates 15 kW of power for the reference point plasma density of $7.5 \times 10^{11}/\mathrm{m}^3$ and motional electric field of 0.18 V/m. This is 10 A into an assumed 150-Ω load (Z in Fig. 3). The wire collects electrons over 1.7 km for the reference point conditions, which corresponds to a collecting area of 39 m^2. The magnetic drag, given by the product of the average current in the tether and the end-to-end motional tether voltage divided by the orbital speed of the system, is calculated to be 2.25 N.

Taking passive sphere electron collection to be twice the Parker–Murphy limit, we can define an equivalent ball system 1, such that it also generates 15 kW of power with a 10-km tether at the reference point conditions (assuming in addition an electron thermal energy of 0.15 eV and a magnetic field of 0.3 G) and the same 150-Ω load. This equivalent ball turns out to be quite large. Its radius is 8.3 m, so that it is roughly the size of a five-story building. Its area is 872 m^2 (over 20 times greater than the wire's collecting area at the reference point). The mass, drag, and the operational difficulties that deploying and maintaining such a large system would entail make it an implausible equivalent in our opinion.

Another approach would be to take an equal-area sphere. As we have seen, the wire's collecting area is not fixed. Rather than limiting the equivalent sphere to the wire's collecting surface at our reference density, let us give it the area on which the wire would collect electrons when the plasma density decreases by a factor of 10. The collecting length then grows to 6.1 km. Equivalent ball system 2 is then defined to have the same collecting area as a 6.1 km length of the reference tether. The ball radius is then 3.4 m. To achieve 15 kW, the tether for this system has to be 89 km long under the reference point conditions, and the corresponding magnetic drag is found to be around 20 N, which would effectively rule it out of consideration.

When we look at what happens as the systems pass into darkness, or have to operate at higher altitudes, where plasma densities may drop by a factor of 10 or more, we find the bare tether further demonstrates its superiority to the standard passive ball collector. At first glance Eq. (2) might appear to be saying that the current is linear in N_e. However, the following analysis shows why this is not true. We begin by writing Eq. (2) as

$$I = \tfrac{2}{3} C_{\mathrm{OML}} (N_e/E)(EL - IZ)^{\frac{3}{2}} \tag{3}$$

The efficiency of orbital to electrical energy conversion $\varepsilon_{\mathrm{OE}}$ is the power to the load ZI^2 divided by the rate of orbital energy loss, which is equal to the average current flowing in the tether times the end-to-end motional voltage EL. The current in the tether below the point of zero bias is just I by definition. We can calculate the average current above the point of zero bias in terms of I by using Eqs. (1) and (2). The overall average current in the tether is then found to be $I(\frac{2}{5} + \frac{3}{5}IZ/EL)$, and we have $\varepsilon_{\mathrm{OE}} = IZ/EL(\frac{2}{5} + \frac{3}{5}IZ/EL)$. We consider the case where $\varepsilon_{\mathrm{OE}}$ is near unity, which occurs for $EL \approx IZ$.

The current will clearly have to decrease if the electron density decreases. However, a decrease in the current I implies both an increase in the tip voltage and the collecting length, as IZ decreases. Thus, the zero bias point moves down the tether, as shown in Fig. 5. Any decrease in I must bring a corresponding increase in the factor in Eq. (3) with the $\frac{3}{2}$ power. If IZ is comparable to EL, which is true in the high-efficiency case we are considering, this factor can largely offset the decrease in density.

From Eq. (3) we obtain the following expression for the derivative of the current with respect to the plasma density:

$$\frac{\mathrm{d}I}{\mathrm{d}N_e}\left(1 + \frac{3}{2}\frac{IZ}{EL - IZ}\right) = \frac{I}{N_e} \tag{4}$$

The condition of high efficiency allows us to ignore the first term in parentheses in Eq. (4), which then enables us to eliminate a factor

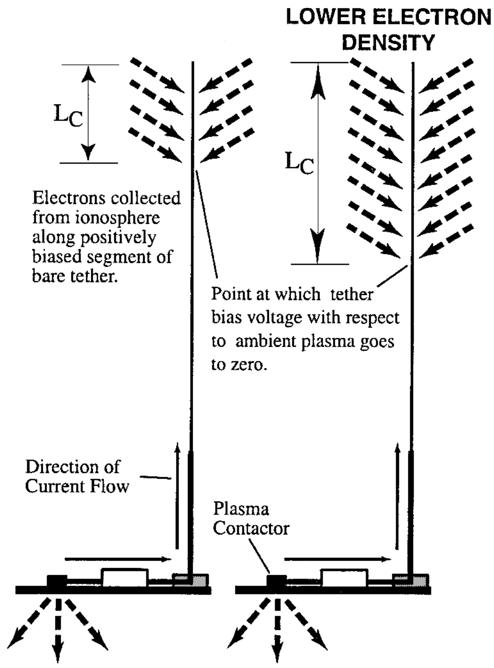


Fig. 5 Adjustment of bare-tether generator to plasma density decrease.

I on each side of the equation. Then, noting that EL is a constant, we can write the convenient equation

$$\frac{\mathrm{d}(EL - IZ)}{\mathrm{d}N_e} = -\frac{2}{3}\frac{EL - IZ}{N_e} \tag{5}$$

Equation (5) is easily solved to obtain

$$\frac{EL - IZ}{EL - I^0 Z} = \left(\frac{N_e^0}{N_e}\right)^{\frac{2}{3}} \tag{6}$$

in which the zero superscripts indicate the original (known) values for which we have obtained an exact solution. We can then write Eq. (6) as

$$I = (EL/Z)\left[1 - \left(N_e^0/N_e\right)^{\frac{2}{3}}\left(L_C^0/L\right)\right] \tag{7}$$

by making use of $EL - I^0 Z = EL_C^0$.

Equation (7) may be taken as a good approximation to how the current collected by a system based on a bare tether varies with plasma density so long as N_e does not vary so much from its starting point that it takes the current out of the assumed region of high efficiency. Because of the $\frac{2}{3}$ power variation in the density ratio, the condition of high efficiency can still be maintained for a relatively large drop in plasma density.

When we carry through the same analysis for a spherical collector, assuming a $V^{1/2}$ law for collection, we arrive at

$$I = (EL/Z)\left[1 - \left(N_e^0/N_e\right)^2(1 - \varepsilon)\right] \tag{8}$$

This shows that the falloff with N_e is much more rapid in the case of the sphere. The current of Eq. (8) will clearly run out of the high-efficiency regime with relatively small decreases in N_e. We note that the only real difference in Eqs. (7) and (8) is in the exponent of the density ratio because $(1 - \varepsilon_{\mathrm{OE}}^0) \approx L_C^0/L$ for the bare-tether generator. Obviously, the power law of current variation with bias voltage at the upper end is the decisive factor in determining the behavior under density variations.

Figures 6 and 7 compare the performance of the 10-km bare-tether system and the equivalent ball system 1 already considered (equal tether length, larger collecting area) under plasma density variations of the sort that can be encountered in a single revolution in low Earth orbit. The motional voltage is held constant. The bare-tether system's current is calculated numerically from Eq. (3), and the current for the spherical-collector-based system is calculated from the equivalent (zero tether resistance) collection equation based on twice the Parker–Murphy collection limit. The power generated, which is proportional to the square of the current, is shown in Fig. 6, and the efficiency of energy conversion, which is nearly proportional to the current, is shown in Fig. 7.

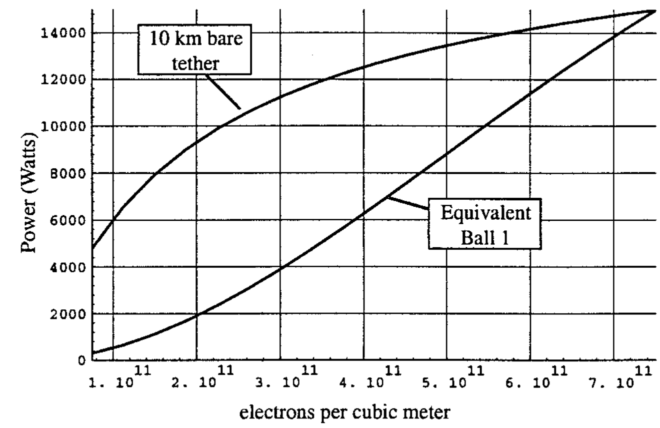


Fig. 6 Variations in generated power with N_e for ideal 10-km bare-wire system and equivalent (equal tether length) ball collector.

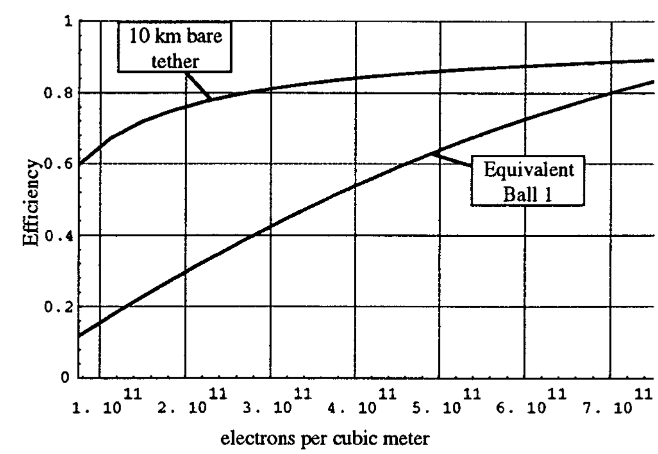


Fig. 7 Variations of $\varepsilon_{\mathrm{OE}}$ with N_e for ideal 10-km bare-wire system and equivalent (equal tether length) ball collector.

The motional electric field component $E = \boldsymbol{v} \times \boldsymbol{B} \cdot \hat{\boldsymbol{\ell}}$, which provides the voltage that collects the current, also varies around an orbit. We now consider how variations in E affect the current collection. As before, we assume we are in the high-efficiency regime to start, that is, plasma density is sufficient. Then the derivative of the current with respect to E can be found from

$$\frac{\mathrm{d}I}{\mathrm{d}E}\left(1 + \frac{3}{2}\frac{IZ}{EL - IZ}\right) = I\frac{EL - IZ + \frac{3}{2}EL}{E(EL - IZ)} \tag{9}$$

which follows from Eq. (3). Using the same techniques as were used to get Eq. (5) from Eq. (4), we obtain

$$\frac{\mathrm{d}I}{\mathrm{d}E} \approx \frac{L}{Z}$$

which applies so long as the efficiency is near unity. Then we have $I/E \approx I_o/E_o$ for decreasing E, so long as $E \gg (1 - \varepsilon_{\mathrm{OE}}^0)E^0$.

The good news out of this result is that, if we are converting orbital energy to electrical at high efficiency, we can maintain good efficiency even with large decreases in the motional electric field, for example, by a factor of 3 for $\varepsilon_{\mathrm{OE}}^0 = 0.9$. However, the power will decrease roughly with the square of the motional electric field. There is no cure for this problem because the electric field is our energy source. The bare tether is no different from a passive sphere tethered system in this respect. We can boost the power, at the expense of efficiency, by decreasing the load impedance. Thus, a variable impedance system is required to maximize orbital average power

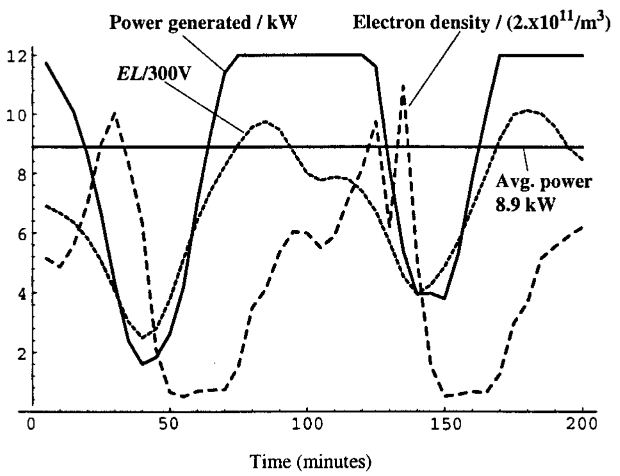


Fig. 8 Power generated by a bare-tether generator as it encounters varying E and N_e around an ISS-type orbit.

and to minimize power variations. The tradeoff is average power vs efficiency, and so system design must take into account which is more important, keeping in mind that lower efficiency means higher magnetic drag, which must be compensated for to avoid orbital decay.

Figure 8 shows an example of how the combined effects of plasma density and motional voltage variations would affect a real bare-tether generator. Here numerical calculations have been made using the full equations of Ref. 1, with tether resistance included. The tether considered is made of aluminum and is 18 km long. The tether geometry is that of a tape 0.7 × 11 mm. The impedance is varied to keep the maximum instantaneous power below 12 kW, though higher powers (at lower efficiency) could be reached, assuming plasma contactors could handle the higher currents that would be collected. The impedance is lowered to achieve maximum power when troughs in E are encountered.

In line with our approximate calculations, the electric field is seen largely to determine performance. This is a near-worst-case example, with troughs in E and N_e overlapping, but density variations are clearly a secondary effect.

Thruster Mode

The potential application of EDT that has drawn the most interest recently is their use for propellantless reboost of the International Space Station or for orbit raising.[13] In either case, the (partially) bare tether is deployed downward and biased positively with respect to the plasma by means of a power supply. Thus, a tether-based system is a type of electrical propulsion system. Electrons are collected along a portion of the exposed metallic wire. In contrast to the case of the power generator, the maximum bias voltage occurs at the end of the insulation and decreases as we move downward toward the tip of the tether. Despite this difference, the analysis of the system yields results that are analogous to those we have already obtained for the power generator. The thrust comes from the action of the magnetic field on the current in the wire. The general setup is shown in Fig. 9.

The voltage diagram in Fig. 10 contains the basic physics of the bare-tether thruster operation. The vertical axis displays voltages, and the horizontal axis represents distance along the downwardly deployed tether. As before, voltages are in the tether rest frame. At the upper end of the tether (far left in Fig. 10) a hollow cathode maintains the deployment platform (station) at the local plasma potential. The plasma potential increases linearly with distance down the tether (moving to the right in Fig. 10), reaching a maximum positive value of EL at the lower tip of the tether.

A comparison of the voltage diagram in Fig. 10 with the corresponding Fig. 3 for the power generator reveals how the two modes of operation differ. The main difference is that the motionally induced voltage must be overcome by a supplied voltage at the platform to drive a current in a direction opposite to the natural one.

As before, we ignore the ohmic voltage drop in the tether. We assume a constant input power P to drive the tether current. The

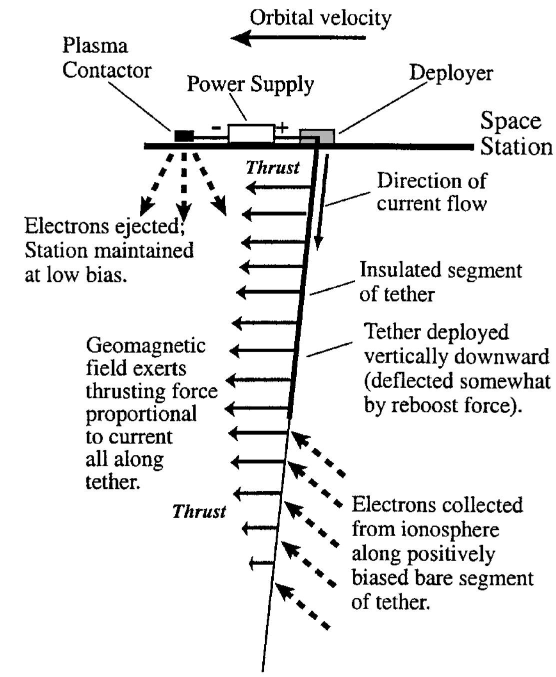


Fig. 9 Schematic of a bare-tether thruster for the ISS.

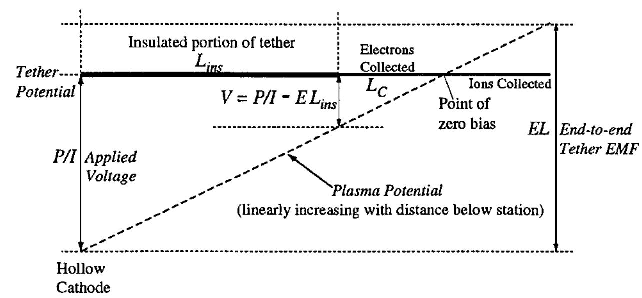
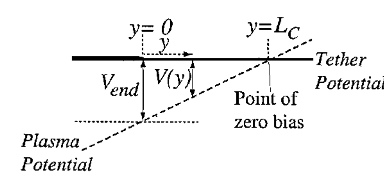


Fig. 10 Voltage diagram for an ideal bare-tether thruster as seen in tether rest frame.

Fig. 11 Voltage diagram for bare-tether thruster in region of positive tether bias.



tether is at a positive potential P/I with respect to the station, with I the current delivered by the tether to the station. Thus, the tether is positively biased with respect to the local plasma as we move down the tether until we reach a point of zero bias. For reasons that will be discussed later, the tether needs to be insulated for a certain length L_{ins} of the upper (attached) portion. To collect a current, the supplied voltage must be greater than EL_{ins}. It is along the bare segment of positive tether bias, designated L_C in Fig. 10, that electrons are collected. From there on out to the lower tip, the tether is negatively biased.

To obtain the total electron current collected by the tether, we apply the basic equation of OML collection [Eq. (1)] to the situation shown in Fig. 11. Despite the different source of the bias voltage in the two generator and thruster cases, the integrals for the current in the two cases are completely analogous with the V_{tip} of the generator replaced by the bias voltage at the end of the insulation $V_{\text{end}} = P/I - EL_{\text{ins}}$. This is clear when Fig. 11 is compared with Fig. 4.

Integrating over the collecting length, as in the case of the generator, and assuming that the zero point bias occurs somewhere on the tether, we obtain for the current in the insulated part of the tether

$$I = \tfrac{2}{3} C_{\text{OML}} (N_e / E) V_{\text{end}}^{\frac{3}{2}} \qquad (10)$$

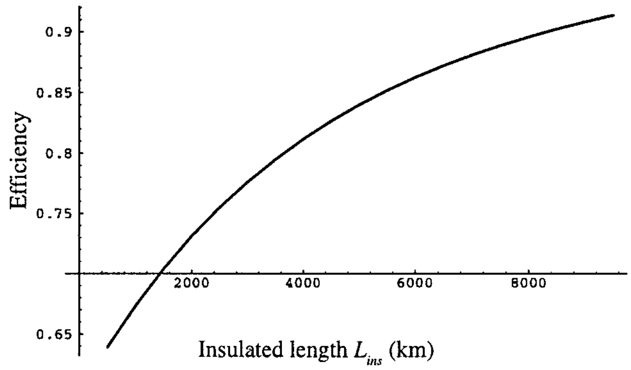


Fig. 12 Efficiency ε_{EO} of a 10-kW bare-tether thruster as a function of insulated length.

The thrusting power generated by the system, as a result of the magnetic force on the tether current, is $IEL_{ins} + \frac{2}{5}IV_{end}$, where we have made use of the easily computed expression $\frac{2}{5}(IV_{end}/EL_C)$ for the average current in the electron-collecting segment of the tether. Thus, the efficiency of electrical to mechanical energy conversion may be written as

$$\varepsilon_{EO} = \tfrac{2}{5} + \tfrac{3}{5}(I/P)EL_{ins} \tag{11}$$

so that the high efficiency condition is $EL_{ins} \approx P/I$.

The magnetic thrust force on the tether is proportional to the integral of the current along the tether. Lengthening the insulated section forces current to flow over a longer portion of the tether. Although the current reaching the upper platform decreases as the insulated length increases (for constant input power), the integral of the current along the tether (and thus the force) increases. Thus, the efficiency of electrical to mechanical energy conversion increases with the insulated tether length. This increase in efficiency is shown in Fig. 12 for the case of a 10-kW power source in a motional electric field of 0.2 V/m. The assumed tether and plasma density are such that 8 A are collected when $L_{ins} = 5$ km. The design of a tether thruster has to balance the need to keep tether mass low, the increased efficiency that comes with greater insulated length, and the necessity for having sufficient bare tether available to collect current under conditions of reduced plasma density.

As in the case of the generator, provided the system has been designed with a bare portion that is sufficiently long, the bare-tether reboost system can offset to a degree the effect of lower plasma densities, by automatically extending the portion of the bare wire on which electrons are collected (Fig. 13). The bias voltage at the end of the insulated portion of the tether (and the collecting length) increase as the current drops, which happens when the plasma density decreases. Again there is a factor raised to the $\frac{3}{2}$ power in the current equation that increases as the current decreases:

$$I = \tfrac{2}{3}C_{OML}(N_e/E)(P/I - EL_{ins})^{\frac{3}{2}} \tag{12}$$

It is more convenient to work with the input voltage $V_{in} = P/I$, which satisfies

$$V_{in} = \frac{3}{2}\frac{PE}{C_{OML}N_e}(V_{in} - EL_{ins})^{-\frac{3}{2}} \tag{13}$$

Restricting ourselves to the high-efficiency regime and proceeding in a way analogous to that followed in the case of the power generator mode, we then obtain from Eq. (13)

$$\frac{d(V_{in} - EL_{ins})}{dN_e} \approx -\frac{2}{3}\frac{V_{in} - EL_{ins}}{N_e} \tag{14}$$

Equation (14) has the approximate solution

$$V_{in} = EL_{ins} + V_{end}^0\left(N_e^0| \; N_e\right)^{\frac{2}{3}} \tag{15}$$

in the high-efficiency region, which can be written as

$$I \approx (P/EL_{ins})\left[1 - \left(N_e^0| \; N_e\right)^{\frac{2}{3}}\left(L_C^0| \; L_{ins}\right)\right] \tag{16}$$

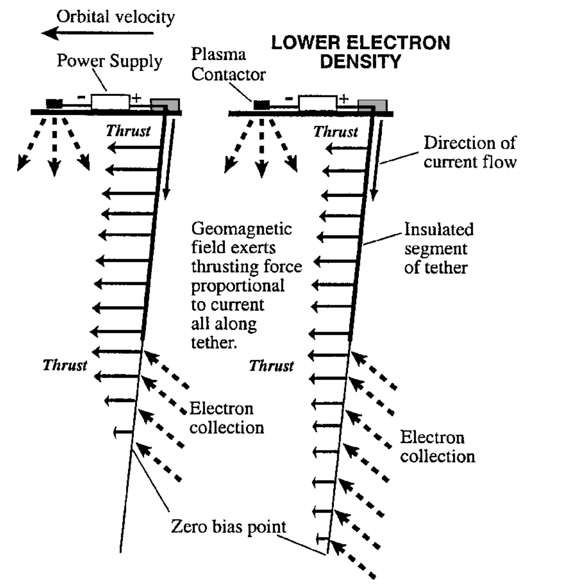


Fig. 13 Adjustment of bare-tether thruster to plasma density decrease.

which, not surprisingly, is very similar to the result found for the generator because the energy source is constant in each case and the current collection equations are the same.

If we carry out the same analysis for a ball collector at the end of an insulated tether of length L, we obtain

$$I \approx (P/EL)\left[1 - (V_0/EL)\left(N_e^0| \; N_e\right)^2\right] \tag{17}$$

where V_0 is the original bias voltage of the sphere. This will quickly violate the condition of the high-efficiency approximation (namely, $I \approx P/EL$) as N_e decreases.

Now we consider variations in E. The situation is different from that of the generator mode, where E drives the current. Here, E works against the current, and a lower E means a higher current for constant input power, as there is a lower voltage to overcome.

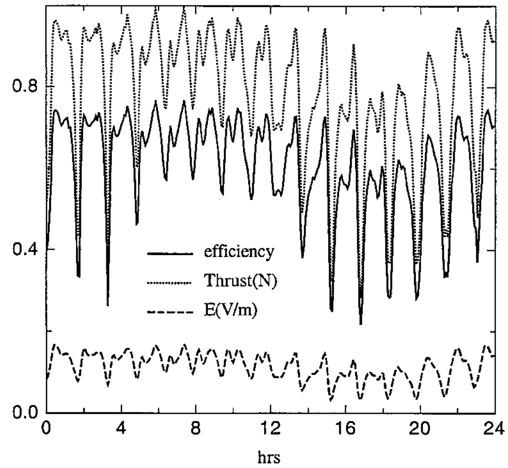
The derivative of the input voltage V_{in} with respect to E is, in the high-efficiency region, given approximately by

$$\frac{dV_{in}}{dE} \approx L_{ins} \tag{18}$$

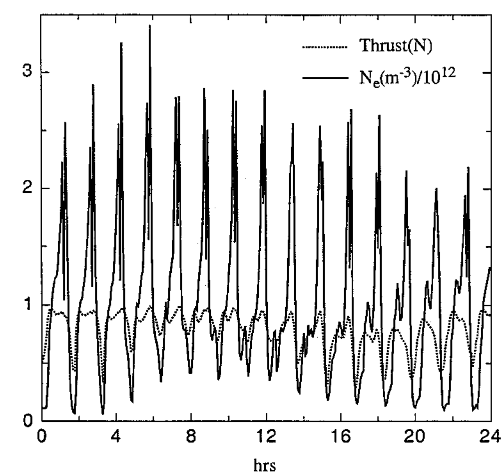
Equation (18) implies $I^0E^0 \approx IE$, so long as the efficiency $\varepsilon_{EO} \approx 1 - \frac{3}{5}(V_{end}^0/EL_{ins})$ is not far from unity. The efficiency (and thrust) decrease with decreasing E, but slowly. We also found a steady efficiency under E variations in the power generator case, but the consequences were quite different there because E was the energy source.

A bare-tether thruster, designed for high efficiency, has been shown to generate a steady thrust under variations in E and N_e, so long as the deviations are not too large. Now let us turn to a real-world system, applying once again the full equations of Ref. 1. Figures 14a–14c show the operation of a system that might provide reboost for the ISS, utilizing 5 or 10 kW of the station's solar power over a 24-h period. The orbit is a possible one at the inclination and altitude of the ISS orbit. The tether is similar to the one whose performance was shown in Fig. 8 in the power generator case. It is an aluminum tape of width 1 cm and thickness 0.5 mm. The tether is 10 km long, and the insulated portion of is 7 km long. The wire's resistance is taken to be a constant 57 Ω. The hollow cathode is assumed to keep the station at a potential of 30 V with respect to that of the ambient plasma. The system does not truly operate in the high-efficiency regime (average efficiency is around 0.71) as assumed in our calculations, but it is close enough to see the benefits of bare-tether system self-adjustment. The thrust is found to vary only by a factor of 2.7 in the 5-kW case and 3.5 in the 10-kW case, whereas N_e varies by a factor of 54 and E varies by a factor of 5.1.

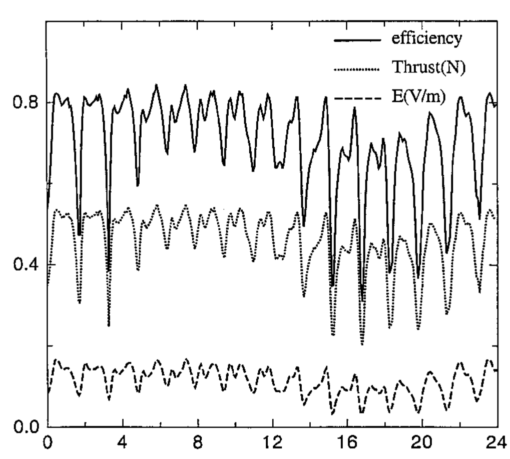
Comparing Figs. 14a and 14c, we see that doubling the power nearly doubles the thrust, while slightly reducing the efficiency of



a) ε_{EO}, thrust, and E for 10-kW input



b) Thrust and N_e for 10-kW input



c) ε_{EO}, thrust, and E for 5-kW input

Fig. 14 Variations around 24-h of an ISS-type orbit.

energy conversion. As might be expected, the thrust generated is roughly proportional to the input power. Thus, the same tethered system could run at an input power of 5–10 kW, depending on the circumstances, giving it greater flexibility as compared with any system requiring a fixed operating power.

Conclusions

Reasonably sized, relatively simple electrodynamic tethered systems based on bare tethers should collect currents in the 10-A range due their ability to collect current in the OML regime. Another consequence of the OML collection law when applied to a bare tether in an EDT circuit operating efficiently is that plasma density variations become significantly less important than for systems based on large passive spherical collectors. This should enable both power generator and reboost systems based on bare tethers to operate night and day because of the self-adjusting collecting area inherent in the system. The strength of the magnetic field and its orientation with respect to the system's velocity vector (which determine the component of motional electric field along the tether) mainly determine power variations for any EDT power generator, though high-efficiency operation can be maintained by a bare-tether system if variations in power are acceptable. A bare tether operating as a thruster at constant input power with high efficiency should maintain a fairly steady thrust even with wide variations in motional electric field and plasma density. The ProSEDS experiment should be a good test of whether OML current can be collected by a wire moving at orbital speeds through the ionosphere, but it will not be a test of a practical system, which will have to come later.

Acknowledgment

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References

[1]Sanmartín, J. R., Martínez-Sánchez, M., and Ahedo, E., "Bare Wire Anodes for Electrodynamic Tethers," *Journal of Propulsion and Power*, Vol. 9, No. 3, 1993, pp. 353–360.

[2]Johnson, L., and Ballance, J., "The Propulsive Small Expendable Deployer System (ProSEDS) Experiment," *Proceedings of the Tether Technology Interchange Meeting*, NASA CP-1998-206900, 1998, pp. 103–108.

[3]Stone, N. H., and Bonifazi, C., "The TSS-1R Mission: Overview and Scientific Context," *Geophysical Research Letters*, Vol. 25, No. 4, 1998, pp. 409–412.

[4]McCoy, J., O'Neill, C., Stanley, J., Settecerri, T., Grossi, M. D., Estes, R. D., Dobrowolny, M., Vannaroni, G., Melchioni, E., Bonifazi, C., Cosmovici, C., Iess, L., Jost, R. J., Olsen, R. C., Ferguson, D. C., Tolbert, R., Rau, D., Katz, I., Lilley, J., Carroll, J. A., Tacconi, G., Mina, L., and Goree, W., "Plasma Motor-Generator (PMG) Flight Experiment Results," *Proceedings of the Fourth International Conference On Tethers in Space*, Smithsonian Inst., Washington, DC, 1995, pp. 57–82

[5]Gilchrist, B. E., Bonifazi, C., Bilen, S. G., Raitt, W. J., Burke, W. J., Stone, N. H., and Lebreton, J. P., "Enhanced Electrodynamic Tether Currents due to Electron Emission from a Neutral gas Discharge: Results from the TSS-1R Mission," *Geophysical Research Letters*, Vol. 25, No. 4, 1998, pp. 437–440.

[6]Parker, L. W., and Murphy, B. L., "Potential Buildup on an Electron-Emitting Ionospheric Satellite," *Journal of Geophysical Research*, Vol. 72, No. 5, 1967, pp. 1631–1636.

[7]Thompson, D. C., Bonifazi, C., Gilchrist, B. E., Williams, S. D., Raitt, W. J., Lebreton, J. P., Burke, W. J., Stone, N. H., and Wright, K. H., Jr., "The Current–Voltage Characteristics of a Large Probe in Low Earth Orbit: TSS-1R Results," *Geophysical Research Letters*, Vol. 25, No. 4, 1998, pp. 413–416.

[8]Cooke, D. L., and Katz, I., "TSS-1R Electron Currents: Magnetic Limited Collection from a Heated Presheath," *Geophysical Research Letters*, Vol. 25, No. 5, 1998, pp. 753–756.

[9]Vannaroni, G., Dobrowolny, M., Lebreton, J. P., Melchioni, E., De Venuto, F., Harvey, C. C., Iess, L., Guidoni, U., Bonifazi, C., and Mariani, F., "Current–Voltage Characteristics of the TSS-1R Satellite: Comparison with Isotropic and Anisotropic Models," *Geophysical Research Letters*, Vol. 25, No. 5, 1998, pp. 749–752.

[10]Alpert, Y. L., Gurevitch, A. V., and Pitaevskii, L. P., *Space Physics with Artificial Satellites*, Consultants Bureau, New York, 1965, p. 205.

[11]Chung, P. M., Talbot, L., and Touryan, K. J., *Electric Probes in Stationary and Flowing Plasmas: Theory and Application*, Springer–Verlag, New York, 1975, pp. 10–13

[12]Sanmartín, J. R., and Estes, R. D., "The Orbital-Motion-Limited Regime for Cylindrical Langmuir Probes," *Physics of Plasmas*, Vol. 6, No. 1, 1999, pp. 395–405.

[13]Estes, R. D., Lorenzini, E. C., Sanmartín, J., Peláez, J., Martínez-Sánchez, M., Johnson, C. L., and Vas, I. E., "Bare Tethers for Electrodynamic Spacecraft Propulsion," *Journal of Spacecraft and Rockets*, Vol. 37, No. 2, 2000, pp. 205–211.

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